

## **Phase Noise Reduction Factor ( Reviewed 8/28/06 )**

### **R Effect:**

The phase noise reduction effect has been known for many years. Most certainly since WW2. It is used in phase locked loops, servomechanisms, and in a slightly different variation in wideband FM. The principle is well known in data systems with correlation. It can even be said to be a necessary part of DSSS to derive the processing gain.

The concept is simple. If the information required is transmitted over a wide bandwidth, but the actual desired information occupies a much narrower bandwidth, the effect of noise can be reduced.

Examples:

Wideband FM :	Transmitted with deviation ratio of 5/1, ( +- 90kHz ), but the final filtered audio has a 15kHz bandwidth.
Phase Locked Loop:	Used to track a signal in a wide noise bandwidth to yield a signal with a low noise bandwidth.
DSSS	Chip Rate ( broadband )/ Signal BW ( narrow ) is called processing gain. 1.25 Mb/s chip rate and 12.5 kHz audio filter yields a "processing gain" of 100/1
Correlation	A number of IF cycles after detection are integrated to result in a narrower bandwidth than the original IF bandwidth.
M-ary-OFSK	A broad bandwidth is required for transmission, but the individual symbols are detected in a narrow Nyquist bandwidth.

### **The Nature of Signal Plus Noise in MSB:**

The swept bandpass of the IF filter is shown in the appendix in Fig. A1. This filter passes only the single frequency of the MSB signal plus any noise that can pass through the narrow passband of the filter. The MSB signal is a single frequency bearing abrupt phase changes which are passed, since the filter has no group delay. This means that the noise can have abrupt phase changes as well and they will also be passed by the filter.

Signal plus noise in the MSB case is two vectors, one for the signal and one for the noise, at nearly the same frequency. If the limiter has little or no 2<sup>nd</sup> harmonic distortion, they will not mix in the circuitry, but will be vector additive at the filter output. If the limiter range is adequate, no amplitude level changes will survive the limiter and only phase changes will remain. Since there is near zero group delay in the filter, both signal and noise can have instantaneous phase reversals.

The Phase Locked Loop can be used to introduce a "Phase Noise Reducing" ( R effect ) by causing the PLL to track the low frequency noise phase changes as it generates a reference phase for a phase difference detector.

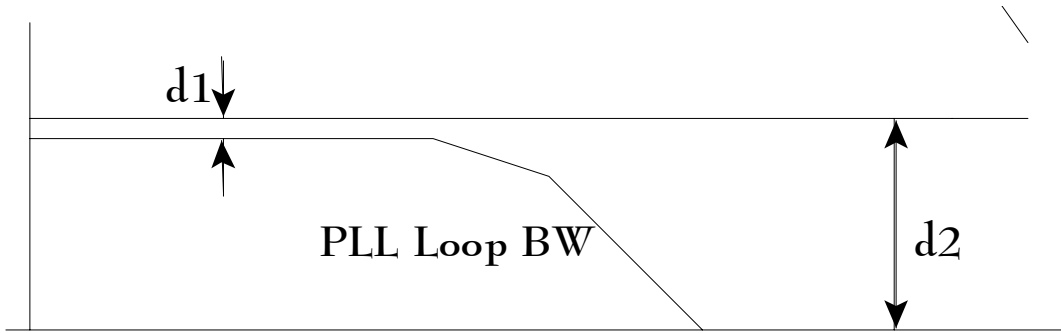


Fig. 1.

Figure 1 shows a PLL tracking a noisy signal to result in a relatively phase noise free lower frequency bandwidth ( Loop Bandwidth ). A wide open input will extend far to the right side. There will be some error voltage as shown by d1 because the tracking is seldom perfect. A phase difference detector will have an output level = d1. The arrow d2 indicates the power and large tracking error that exists for the full bandwidth of the phase detector, where the PLL tracking ability is lost. A PLL will have a loop filter that limits the upper limit of the phase change slew rate that can cause a tracking action by the loop. This method is used with 3PRK and 3PSK. The loop time constant is dozens of bit periods. The purpose is to obtain a reference for the detector that is as free of noise effect as possible

The "phase noise reduction factor " R" used to improve the SNR is given by Best (1) as an improvement over the full bandwidth SNR.

--With no reduction:

$$SNR_{input} = \frac{P_s}{P_n} = C/N$$

Signal Power/Noise Power, or in RF terms C/N ( carrier power to noise power ) that applies for any 2 level digital system.

If FM or PM are involved, a modulation angle correction  $\beta = \sin \theta$  is added. The rule for the Nyquist bandwidth is that it is equal to the bit rate, which equals the sampling rate.

Best (1) then proves mathematically that (BR)/(2Loop Filter BW), or (Signal BW)/(2Loop Filter BW) is the phase noise reduction factor when a PLL is used.

$$SNR = \frac{P_s}{P_n} \frac{B_i}{2B_{Loop}} = \beta^2 \left( \frac{Bitrate}{2FilterBW} \right) \left( \frac{SignalPower}{NoisePower} \right)$$

For a double sideband RF method, the 2 is omitted and Filter BW becomes the Nyquist BW. With MSB/VMSK, the zero group delay filters pass the full noise bandwidth = Intermediate Frequency. The R effect must be obtained somewhere from integration, as in the loop BW of a PLL in the detector. If the bit rate and filter Nyquist BW are the same, there is no SNR improvement.

Referring to Fig. 1, the MSB signal passed through a zero group delay filter can and must be detectable as a single cycle phase change (  $T \sim$  ). This means the upper limit of the frequency to be passed is the IF frequency = Nyquist Bandwidth. This is also the detector sampling rate in a synchronous detector, or phase detector.

A conventional PLL will track low frequency shifts up to the upper limit of the loop Bandpass. This is not sufficient bandwidth to detect the change of one or two cycles out of the data stream. The output desired from 3PRK or 3PSK MSB is a very narrow pulse, not a lower frequency. **The R effect is negated when only one or two IF cycles are used.**

The reference recovery solution for 3PRK and 3PSK is to use a phase difference detector that detects the difference between the phase of the PLL tracking ( reference ) oscillator and the incoming signal. Assume the PLL loop high frequency roll off will track out almost to the highest frequency to be passed as in Fig. 1. Any low frequency phase differences passing the bandpass filter after vector summation will be tracked with a small phase tracking error  $d1$ . Above the roll off, the phase tracking error ( signal response ) is as large as 180 degrees, shown as  $d2$ .

The result is a phase difference detector that tends to remove low frequency phase noise from the reference, but keep any high frequency components for the detector. The abrupt phase changes of MSB/VMSK signals are just like the high frequency phase noise.

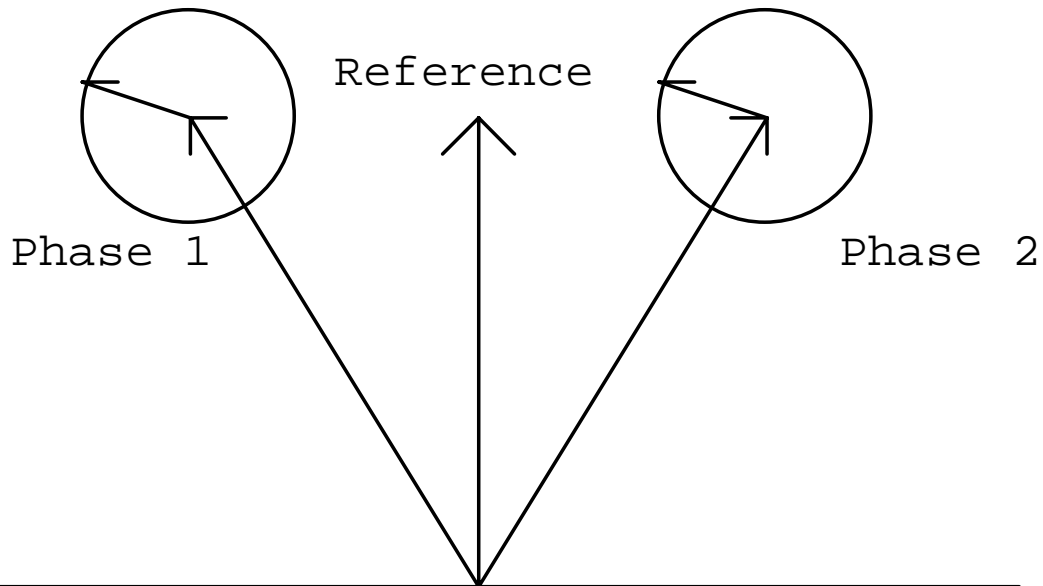


Figure 2.

The desired signal shifts from phase 1 to phase 2. Interference will cause a shift in position at the ends of the swing. For 5 channels extending  $\pm 60$  kHz, the reference will also tend to track the interference, since the phase shift is at 60 kHz or less vs a 3 to 6 Mb/s data shift, and thus reduce its effect. This is true of the PLL, or an LC reference recovery circuit where group delay replaces PLL loop delay. Or LC bandwidth replaces loop bandwidth.

If the interference is too high in frequency for the reference recovery circuit to follow it, the vectors are as seen in Fig. 2. **If the reference circuit can track the interference, the reference vector moves with the interference and the interference has no effect.**

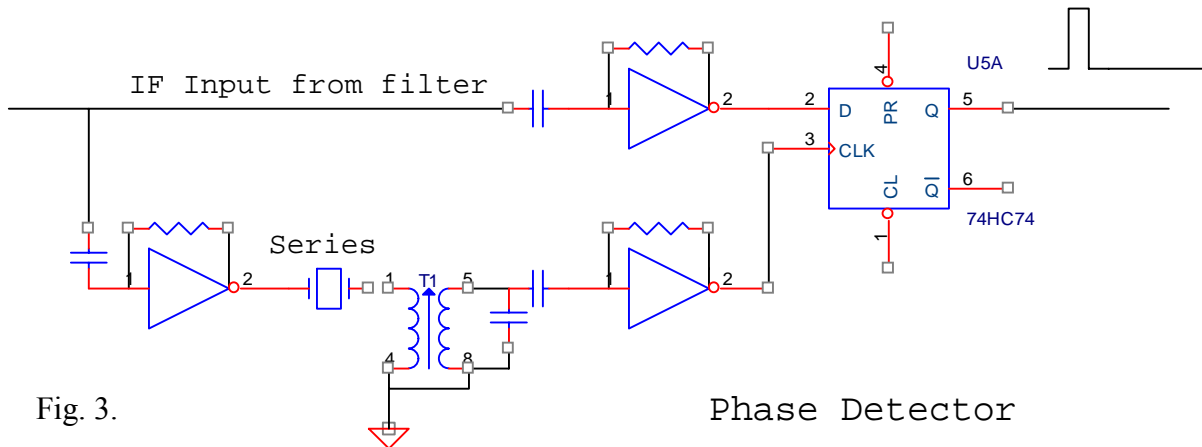


Fig. 3.

Phase Detector

Figure 3 shows the phase difference detector. There are several phase detector devices that can be used. The XOR and the D Flip Flop are both widely used in PLL design. For NRZMSB and VMaxSK, the crystal is not necessary. The transformer Q (bandwidth) determines the slew rate.

The crystal and transformer circuit at the lower portion of Fig. 3 separates the low and high frequency phase noise components. This circuit will have a large group delay, ( equivalent to Loop Filter RC time ), which is traditionally calculated to be:

$$T_g = [\Delta\Phi / (2\pi \Delta f)]$$

For LC or Gaussian filters, this is:

$$T_g = [ 1 / (4\Delta f) ] \quad \text{and} \quad T_g = Q / (4IF)$$

IF = Intermediate Frequency

Obviously, a very narrow  $[\Delta f]$  bandwidth filter, or filter with a high Q, has a very large group delay. This means the crystal and transformer will be very slow in doing any phase tracking.

There is an associated equation for the rise time of the conventional filter, such as the tank circuit in Fig. 3:

$T_r = 0.7/B$ , where B is the 3 dB bandwidth  $[\Delta f]$  of the filter. This is the time from 10% to 90% on the RC curve. It is customary practice to assume  $T_r = 1/B$ .

A traditional PLL will have a RC loop filter with a roll off time constant  $T_r = RC$ . Any amplitude or phase changes passing through the loop filter will be subject to this  $T_r = 0.7/B$  rise time relationship. The loop cannot pass a zero group delay phase change pulse of one cycle, but could pass a phase change rising along an RC equivalent slope, if the Q ( or filter BW ) is adjusted accordingly.

Assume instead of the amplitude rise time, that there is a phase slew rate applicable. This phase slew rate  $\delta\Phi/\delta\tau$  is approximately 160 degrees in the RC rise time, or  $.8\pi$  radians, with the frequency fixed. Given a slew rate, or rise time, which is more than 5 times the IF cycle period (  $1/f$  ), the detector can separate low and high frequency phase noise. The high frequencies passed will include the data portion of an MSB signal.

Gaussian white noise consists of all frequencies within a defined bandwidth. In the MSB case, that BW is the IF Bandwidth. The IF filter and limiter stages will limit the noise power bandwidth coming to the detector, but unfortunately, any phase noise is passed to the phase detector, regardless of the phase change rate, subject to the slew rate.

Any slow phase change will pass the tank, ( equivalent LC of the crystal in Fig. 3 ) following the allowed rise time ( slew rate ), but any very fast change will be unable to overcome the delay. Although this is not a PLL ( Fig. 3 ), the effect is similar. The R effect shown here using a PLL equivalent tracking the low frequency phase shift reduces the amount of phase shift due to low frequency noise seen by the phase difference detector.

Fig. 4 is a plot showing the random frequency characteristics of noise. Note that it consists of high and low frequencies. If the PLL providing the reference has a loop filter BW that can track the lower frequencies 'a', but not the highest ones 'c' 'd' 'e', a phase difference detector ( Fig. 3 ) will have a high level output from the highest frequencies ( most rapid phase changes ), but little or no output from the low frequency phase changes. 3PRK and 3PSK MSB modulation provides the highest possible frequency/phase change, being 1 cycle at the RF or IF frequency.

The most likely noise vector summation is a low frequency wave, with the high frequency noise 'c' riding on top of the wave. Using a PLL to track the low frequency phase noise isolates the high frequency vector summation for the difference detector. The *high frequency noise peak level* is reduced at the reference output as seen at 'd', since it is no longer riding on top of a strong low frequency wave. The error probability is greatly reduced for both low and high frequency noise. The desired signal is also a high frequency phase change, shown at d2 in Fig. 1 and at 'e' in Fig.3. This change is beyond the loop tracking frequency.

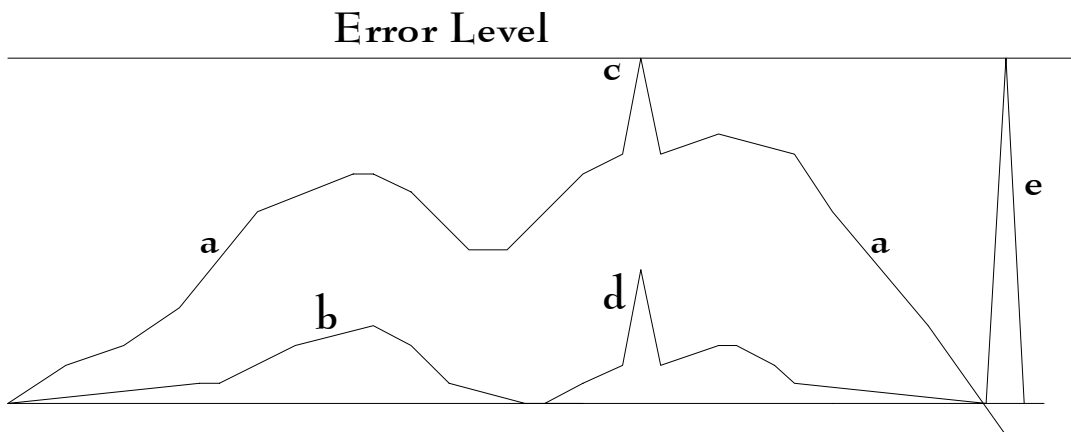


Fig. 4.

If the phase detector reference phase can track the background lower frequency noises seen in Fig. 4, but remain relatively unchanged by a sudden one or two cycle modulation burst, then a tracking relationship can be obtained as seen at d1 in Fig. 1 for the low frequency phase changes. The low frequency phase changes are spread over a large number of IF cycles, which are tracked by the LC. MSB is interested primarily in the

leading edge of the modulation change, which is an abrupt high frequency, or quick phase change pulse, as far as the PLL is concerned. This remains unchanged at d2, or 'e'.

The term Q ( not coil Q ) in statistical analysis expresses the period of time a noise peak will rise above a certain level. It is a plot of periodic peak signal voltage vs RMS noise. For a 2.0/1 Q ratio, the BER is  $2.275 \times 10^{-2}$ . For 4.0/1 it is  $5 \times 10^{-5}$ . Thus, altering the relative level of a high frequency noise spike can reduce the BER dramatically. Removing the low frequency noise reduces the level 'c' to 'd', but leaves the desired pulse change d2 or 'e' unaffected. There will a substantial improvement in BER.

$$P_e = \frac{1}{2} \operatorname{erfc} [\operatorname{SNR}]^{\frac{1}{2}} = \mathbf{BER} = Q = \left( \frac{E_s}{\sigma} \tau \right)^{\frac{1}{2}} \quad \mathbf{Ref. -Bellamy [3]}$$

**There can be a very large R effect for low frequency and very little R effect for high frequency phase noise in the noise bandpass spectrum using the present circuits. The improved SNR greatly improves BER.**

The simple phase reference detector ( Fig. 3 ) will exhibit this PLL effect and can be optimized for the best slew rate. For a given rise time allowing a low frequency phase change to pass,  $T_r = 0.7/B$ , with a slew rate  $\delta\Phi/\delta\tau$  of  $.8\pi$  radians during  $T_r$ . In general analysis, they are assumed to be  $T_r = 1/B$  and  $\pi$  radians. The slew rate is determined by the crystal Q [ 180 degrees in  $T_g = Q/(4IF)$  ]. Drive power is limited by the limiter circuits and thus has little or almost no effect.

Best (1) treats the R effect in terms of the PLL. ( Baseband, not RF ).

$$\operatorname{SNR} = \left( \frac{\operatorname{Bitrate}}{2\operatorname{FilterBW}} \right) \left( \frac{\operatorname{SignalPower}}{\operatorname{NoisePower}} \right) = \left( \frac{\operatorname{Bitrate}}{2\operatorname{FilterBW}} \right) \left( \frac{\operatorname{SignalPower}}{\operatorname{NoisePower}} \right)$$

from Best sect. 3.4. 1 and 3.4.2. ( Eq. 3.82)

**There is an error in this formula as shown. The 2 should be omitted for RF.**

$$R = (\operatorname{BitRate})/(\operatorname{Filter Nyquist BW})$$

He cites as an example - wideband FM - where the R effect greatly improves the SNR of the FM signal, since the bandwidth occupied (equivalent to a bit rate)( assume  $\beta = 5$  ) is greater than the audio filter bandwidth at the output. Best shows how the 4046 PLL Integrated circuit can function as both a bandpass filter and FM detector at 10.7 MHz.

The R factor in broadcast FM improves the SNR by about 7 dB, but the PLL effect also extends the FM knee by about 3 dB. ( See Taub and Schilling Fig. 10.12-2 )

R factor is calculated by T&S differently from the method used by Best. See Eqs. 9.5-8 and 9.5-9.

These effects are apparently applicable to MSB, **but used in reverse**. The complete mathematical analysis is not yet available. The BER measurements will show that the effect is present, though not necessarily optimized.

### Noise Through a Narrow Band Filter:

Any narrow band conventional filter has group delay, which lowers the slew rate and the filter cannot pass rapid phase changes.

*The zero group delay filters do not exhibit this characteristic, while filters with group delay ( PLL Loop Filters and filters with circuit Q ) do exhibit it. This effect can be used to advantage in several modes of VMaxSK and NRZMSB operation*

*See the file on NRZMSB, where the maximum 'R' effect is achieved.*

When broadband Gaussian noise is passed through a narrow bandwidth filter, the result is shown by an illustration from Taub and Schilling [2]. Only a low frequency sum that fits within the bandwidth of the filter is passed. The upper frequency limit is 1/2 the filter BW. This is seen in Fig. 5 (7.5.1).

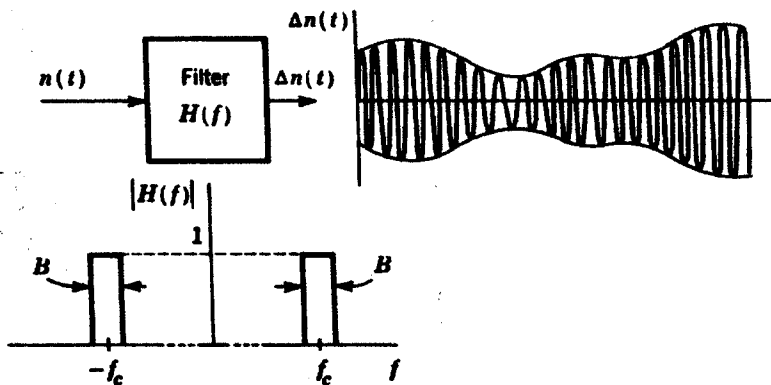


Figure 7.5-1 Response of a narrowband filter to noise.

Fig. 5.

The low frequency that results is much lower than the IF frequency sampled in the detector, or even the bit rate, thus will wipe out a single 1-3 cycle pulse. Also, a **strong** low frequency interference will introduce a large number of missed IF cycles interfering with bit periods. **A true phased locked loop used as a phase detector can work miracles when using this noise changing characteristic along with NRZ modulation as an input.**

Ordinary digital RF signals are sampled at the Nyquist sampling rate, which equals the bit rate, hence only the fraction [ Loop Filter BW]/ [ Bit Rate] of the noise appears in

each sample. **This is the "Phase Noise Improvement Factor" ( 'R' factor ) discussed in Best, Sect 3 [10]** and in more detail below ( Fig. 9 ). It is similar to the 'Processing Gain' in DSSS.. This factor applies to wide band FM as well. See Taub and Schilling Eq. 9.5-9. It applies only when the Nyquist bandwidth is greater than the filter bandwidth.

The theoretical R value for RF is (Bit Rate/FilterBW), but it has an upper limit determined by other factors. If the white noise passes through a narrow bandwidth filter, the output of the filter consists of a low frequency AM and PM signal with its upper frequency limit determined by the bandwidth of the filter. ( See Taub and Schilling,(11) pp 325, Fig. 7.5.1 shown above).

The high sampling rate, combined with the low frequency of the noise, results in only a small portion of the phase noise appearing in each sample. In theory, this can result in a considerable improvement in C/N and SNR.

MSB/VMSK in the 3PRK and 3PSK modes utilize a different filter and phase detector concept, with little or no R effect. In practice, the BER improvements have been limited to about 2-3 dB when using VMSK/MSB modulation and narrow band filters as loop filter equivalents.

**Caution: Warning:** This low frequency effect through a narrow band filter is caused by the group delay (slew rate ) of the filter. The greater the group delay, the lower the noise frequency that can pass. MSB using 3PRK and 3PSK requires a filter with zero group delay, **therefor there is NO low frequency noise effect as described above in the ideal 3PRK system.** HOWEVER, this effect can be introduced at a lower level than calculated from (Bit Rate/IF BW) by making changes in the phase difference detector circuitry. It can also be introduced by using a low group delay pre-filter.

The filter circuit below has no group delay. It responds to each cycle as it is sent, including phase reversals and phase shifts. There are other circuits better for general use.

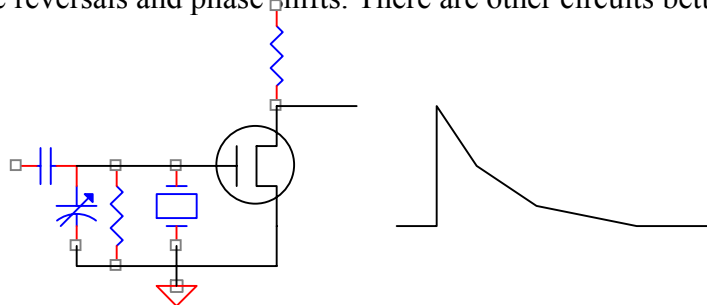


Fig. 6. The Shunt Filter.

## HOW CONVENTIONAL FILTERS WORK:

Conventional filters cannot be used with Ultra Narrow Band Methods.

## THE OPTIMUM FILTER:

The optimum filter is described as "the filter that passes the most signal power with the least noise power". The correlator is considered to be such a filter, although it is also combined in a detector.

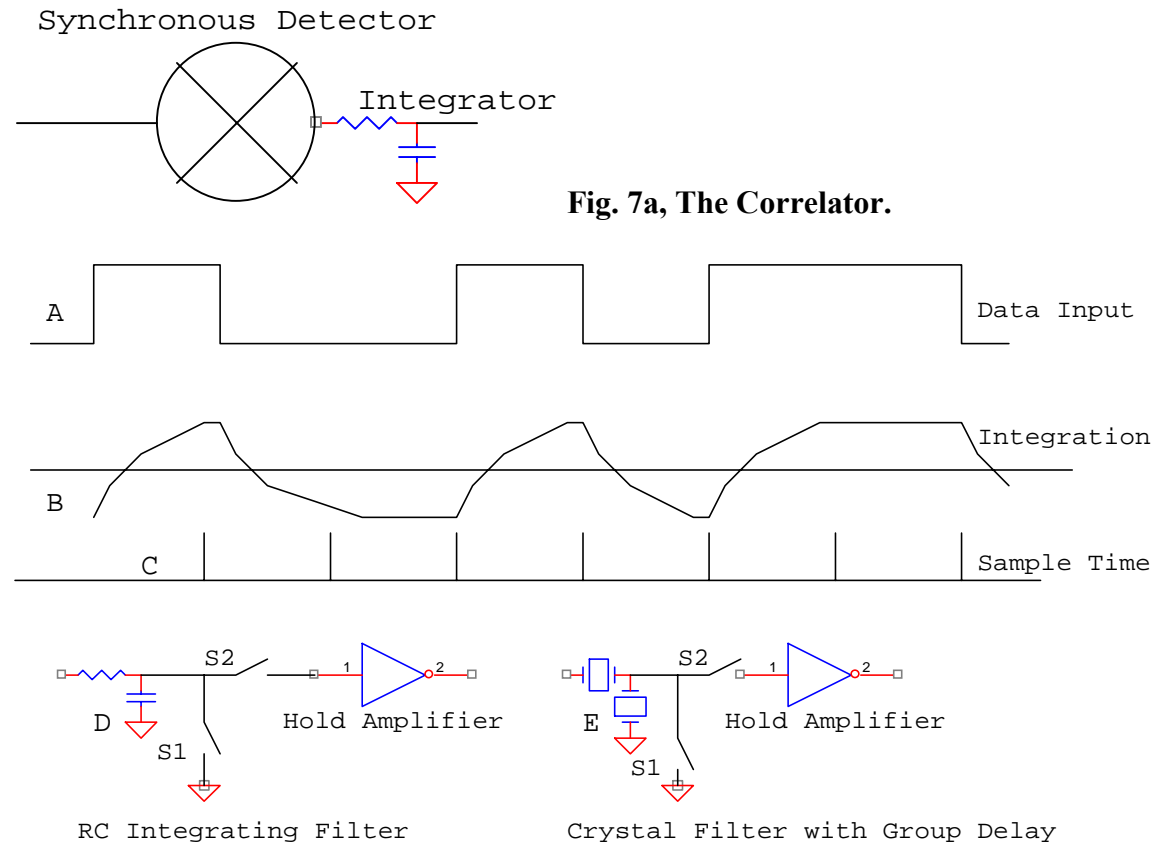


Fig. 7b.

The integrating filter portion of the correlator is shown in Fig. 7b. This filter consists of a sample and hold circuit following an RC integrator. In this case, the integrator RC is optimized for the group delay =  $T_s$ . The group delay of a conventional filter is equivalent to the RC rise time, so a crystal or LC filter can be used if the group delay or rise time is made equal to 1 bit period.

Correlation mathematically is:

$$R(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T v(t)v(t+\tau)dt$$

The integrator is a necessary part of the whole.

$V(t)$  is the detector phase reference and  $v(t + \tau)$  is the signal to be detected. The output is a pulse 1 IF cycle wide. A single pulse as from 3PRK which is integrated destroys the signal, but a series of IF pulses over a bit period will integrate to improve the SNR.

Using the data pattern at (a) as a sample, the integrator charges positively as shown in (b) until it is sampled by S2 at its peak. The peak output after a hold circuit is shown in (c.) The capacitor is then discharged by S1 to be recharged anew by the input signal.

The integrating filter itself is considered to be an “optimum filter” or “matched filter” in the presence of white noise. The maximum signal power is obtained by integrating the incoming pulses. The noise is white and has a long term integrated output level at 0 volts. The signal information will have a positive or negative integrated value.

The input pulse pictured here is considered to be rectangular, but other pulse shapes, such as a series of short pulses, apply as well if the sample time is properly chosen.

**The example above is for amplitude modulation. A similar model using the PLL can made using the loop filter as the signal integrator. The limiter eliminates all AM, leaving only phase noise for the PLL.**

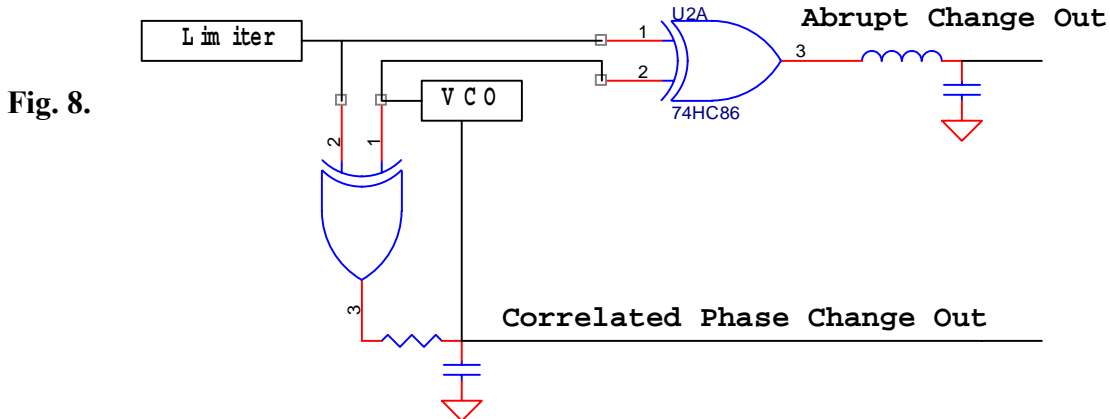


Fig. 8.

The PLL is used to establish a reference frequency that is centered near the ‘I’ reference phase shown in Fig. 9. The abrupt change output can be used for 3PRK and 3PSK. When using NRZMSB, the abrupt phase change output is integrated to obtain the effect.

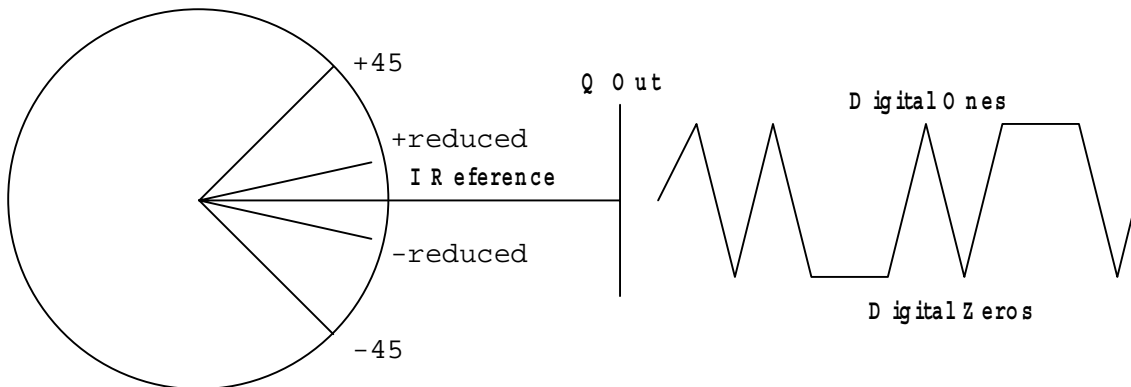


Figure 9. NRZ/MSB modulation with a 90 phase shift will have an output at the phase detector loop filter output that shifts phase with the incoming signal. The tracked output ( correlated output ) is shown at the right. There will be phase losses resulting in a reduced output level from optimum. The data is sampled at the peaks as in the amplitude integrator.

There are two possible modes of operation here. For single pulse detection the loop rise time is made to be in excess of several bit periods. The object is to provide a reference that tracks low frequency noise, yet offers the best reference phase for the detector of an abrupt signal phase change of one cycle duration, as in 3PRK. The second mode has the loop rise time equal to one bit period and the signal phase alterations are obtainable as voltage outputs from the loop filter. In the first mode, there is no signal correlation. In the second mode the signal is correlated. The second mode is applicable to VMSK and NRZMSB, but not to 3PRK and 3PSK.

This circuit takes advantage of the phase noise at the input being changed to low frequency noise, depending on the loop filter bandwidth. The PLL can track the low frequency noise and data using NRZMSB, but the data shifts at a much higher frequency using 3PRK will be detected at the abrupt change output only. The signal at the PLL input is a low frequency noise with the data riding on top of it. The PLL loop bandwidth reduces the low frequency noise effect on each correlating sample.

In figure 9, the correlator sampling rate is at the IF frequency, but the bandpass at the loop filter RC output is made equal to one bit period. A number of IF cycles are integrated to get one data bit. The 'R' effect is optimized in this circuit.

The **matched filter** is best described as the best filter that is usable for the modulation method employed. It may or may not be the optimum filter, since the optimum filter could mask some modulation details.

All conventional filters, LC, Crystal and SAW function on a similar principal. **It is all a matter of phase shift and rise time through the circuit.**

**The 'R' effect can be used to greatest advantage with NRZ or RZ data inputs to an MSB system as shown in Fig. 9. In these cases, the full R Effect comes into play.**

### **Error Probability:**

The error probability for any two phase ( 2 level ) method can be calculated from:

$$P_e = \frac{1}{2} \operatorname{erfc} [\operatorname{SNR}]^{1/2} \quad (\text{Sq. root of SNR is used to get voltage ratio})$$

$$P_e = \frac{1}{2} \operatorname{erfc} [(E_s/\eta)\tau]^{1/2} \quad (\text{energy ratio}) \quad \text{Ref. -Bellamy [6]}$$

$$P_e = \frac{1}{2} \operatorname{erfc} [(E_b/\eta)]^{1/2}$$

$$P_e = \frac{1}{2} \operatorname{erfc} [z] \quad \text{where } z = V_p/1.4E_N \quad (V_p = \text{peak signal level and } E_N = \text{noise RMS})$$

Note the second of these equations. [  $\tau E_{\text{signal}} / \eta$  ], so  $\tau E_{\text{signal}} = E_b$ . = signal power ON for one SubBit period using 3 PRK. This is true energy per SubBit. One SubBit is being detected.

Utilizing an integrator in a post detection circuit, or in the detection circuit, the  $E_b$  can be increased by increasing  $\tau$ .

$$P_e = Q \left[ \frac{A}{\sqrt{Nt}} \right] = Q \left[ \frac{A}{\sigma} \right] = Q \left[ \frac{\text{Cvolts}}{\text{Nvolts}} \right] = \frac{1}{2} \text{erfc} \left[ \frac{A}{\sigma} \right]$$

From Feher [2], where A is the peak signal value at the sampling instant and  $\sigma$  is the RMS voltage of the noise power at the threshold detector input. When using a cycle to cycle comparison, noise peak voltage must be compared to signal peak voltage.

$$Q_{(z)} = \frac{1}{2} \text{erfc} \left( \frac{z}{\sqrt{2}} \right) = \frac{1}{2} \text{erfc} \left( \frac{V}{\sqrt{2N}} \right) = \frac{1}{2} \text{erfc} \left( \frac{E_s}{\sigma} \tau \right)^{\frac{1}{2}}$$

Bellamy, Eq C.19, Rappaport D.11

These equations assume a peak signal voltage V and an RMS noise voltage, which would have peaks at 1.4 times the RMS level. Using a true RMS meter, the relative peak and RMS volts are the same as measured, so the 1.4 correction is not used.

Using a CW interference source, there would be no errors until the noise power is at zero dB. In tests, this is demonstrated to within 1-2 dB.

Using AWG Noise, the Bit Error Rate probability relationship is statistically related to the Q(z) and erfc functions. There will be a noise peak ( according to the Q function ) that exceeds the RMS noise level  $10^{-6}$  of the time when the RMS noise is about 1/4.8 of the signal level, or a 6.8 dB ratio. Unless the R effect can improve on this, the C/N for  $10^{-6}$  BER is at this level. It is known that some R effect improvement can take place by increasing  $\tau$ . See Best [10] and Fig. 9.

The Q(z) curve indicates  $10^{-3}$  BER will occur when C/N = 4.7 dB ( 3/1 voltage ). Any observed values better than these theoretical values are due to the R effect. Q is related to erfc and error probability by the relationship above.

**Note that the value of Q is raised by the square root of  $\tau$ . This results in a dramatic decrease in P(e), one that is not matched by OFSK, which has bandwidth.**

Using a correlating detector with integrating filter, the Q(z) value will be higher using NRZMSB than it is for 3PRK. The P(e) formula is not quite the same as that for OFSK, but the C/N is dramatically reduced below that for BPSK when using MSB and a correlating detector.

The theoretical values for BPSK are about 3 dB worse than for MSB. This is due to several factors. The BPSK measurements are based on an ideal filter, or raised cosine filter, having a bandwidth equal to the data rate ( Nyquist bandwidth ). These conditions do not apply to MSB modulation.

A second relationship is obtained using the 'R' effect according to Best [ 10].

$$SNR = \frac{P_s}{P_n} \frac{B_i}{B_{filter}}$$

$P_s/P_n$  is the original SNR that applies to 3PRK if  $B_i = B_f$ . **(BR/filter Noise BW) is the improvement due to the 'R' factor.** In this case, the filter BW changes when using an integrating detector according to  $N\tau$ . For  $\tau = 10$  ( 10 SubBits integrated to obtain a

correlated data bit ): Then: 
$$Q = \left( \frac{E_s}{\sigma} \tau \right)^{\frac{1}{2}} = P_e$$

The larger the number of pulses  $\tau$  being integrated, the higher the value of  $Q$  and the better the error rate. A very high IF frequency and low bit rate can have an excellent BER.

Best assumed the phase jitter due to noise was  $= 1/SNR$ . This is approximately correct only for small angles. **The correct value is  $\Phi = \text{ARCSIN} = 1/SNR$ .** The maximum possible phase jitter for  $SNR = 1$  is 90 degrees. These equations are for baseband, therefore, the 2 in the denominator above does not apply.  $E_s\tau / \sigma$  is correct.

$$Q = \left( \frac{E_s}{\sigma} \tau \right)^{\frac{1}{2}}$$
 is the correct formula for all ultra narrow band methods.

**When  $\tau$  is equal to the entire bit period, there is a significant reduction in  $E_b/\eta$ .** Using 3PRK,  $E_b/\eta$  closely follows the  $Q(z)$  curve. This is approximately 3 dB better than for theoretical BPSK. Using a correlative detector and large  $\tau$ ,  $E_b/\eta$  can approach 0dB.

**The equivalent loop filter BW is  $1/\tau$ . If  $\tau$  is increased to cover the entire bit period, then  $Q(z)$  is greatly improved.** ( Figs 8 and 9 ).

$$Q_{(z)} = \frac{1}{2} \text{erfc} \left( \frac{z}{\sqrt{2}} \right) = \frac{1}{2} \text{erfc} \left( \frac{V}{\sqrt{2N}} \right) = \frac{1}{2} \text{erfc} \left( \frac{E_s}{\sigma} \tau \right)^{\frac{1}{2}}$$

Bellamy, Eq C.19, Rappaport D.11

$$Q = \left( \frac{E_s}{\sigma} \tau \right)^{\frac{1}{2}}$$
 determines the BER for MSB.  $\tau = 1$  to 3 for 3PRK and MCM.

**The theoretical and measured BER follows the  $Q(z)$  curve.**

$$\frac{E_b}{\eta} = \frac{\frac{\text{SignalPower}}{\text{BitRate}}}{\frac{\text{NoisePower}}{\text{FilterBW}}} = \frac{E_s \tau}{\eta} = C/N$$

With MSB, the highest possible bit rate is the sampling rate, which is the IF Frequency. The near zero group delay filter BW is seen to be = IF Freq. as well. In the above equation, the two cancel leaving C/N.

But if a number of SuperBits are correlated,  $\tau$  increases and  $E_b/N$  improves.

It might be argued, that conventional modulation methods utilizing filters with group delay and coherent correlating detection will also increase the power of  $E_b$  by increasing  $\tau$ . This is correct, but there is also a difference in  $\eta$ . Conventionally, for an optimum system  $BT=1$ .  $B$  = Bit Rate and  $T$  = Bit Period. As the bit rate decreases, the optimum filter noise bandwidth also decreases.

Bit rate and filter BW are tied together. As the filter BW decreases,  $\eta$  increases. The SNR and  $E_b/n$  do not improve in conventional methods.

With MSB,  $\eta$  is fixed at  $(\text{NoisePower})/(\text{IntFreq})$ . It does not change with post detection bandwidth changes ( a filter or PLL with lower loop freq. BW ), so  $E_b/n$  rises with increasing  $\tau$ .

**MSB modulation can be shown to have a lower  $E_b/n$  than OFSK as  $t$  increases.**

$$[(E_s/\eta)\tau]^{1/2} \text{ (energy ratio)} = [(E_b/\eta)]^{1/2}$$

$$E_s \tau = E_b \quad E_s \text{ can be reduced linearly with } \tau \text{ to obtain the same } E_b$$

Ref. -Bellamy [6]

**Shannon's Limit:**

$$R = W \text{ Log}_2 ( 1+C/N ) \text{ or as: } R = (1/\tau) \text{ Log}_2 ( 1+C/N )$$

**It is necessary to understand the meaning of  $W$ .** It is not the filter noise bandwidth used, but the Nyquist bandwidth, which is and must be equal to the sampling rate. One cannot violate the Nyquist sampling theorem. The general practice is to use  $1/(\text{filter rise time})$  as the Nyquist bandwidth =  $(1/\tau)$ . Quoting Schwartz:

**"The system channel capacity 'R' is obtained by multiplying the number of samples per second by the information per sample." ( Schwartz, [1] pp 324 and equation 6-134 ).**

It is obvious from the papers on MSB that the sampling rate is the Intermediate Frequency, or SubBit rate. Thus  $W = \text{Intermediate Frequency for MSB}$ . The noise bandwidth is much less, being equal to the Q of the LC or crystal filter.

It is also obvious that a data rate equal to the Intermediate Frequency could be received and decoded. The actual data rate used is lower, since multiple SubBits are used (integrated) for one data bit. Assume a 48 MHz IF, then

$$48\text{MHz} = 48\text{MHz} \log_2 (1 + C/N)$$

The equation will balance when  $C/N = 1 = 0\text{dB}$ . (Shannon's Limit)

Using a lower R, it appears C/N could be below 0dB as in OFSK.

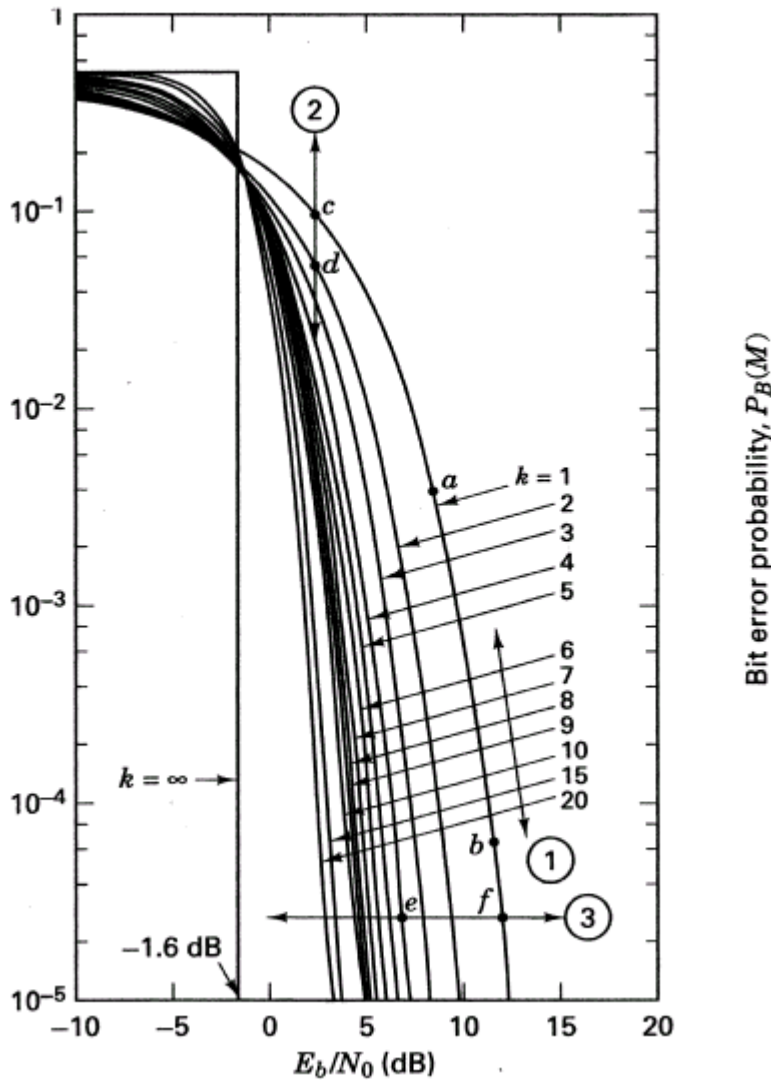


Figure 10. Values for M-ary OFSK. (From Sklar [8]).

$E_b/\eta$  has an asymptotic value of -1.6 dB when M is infinity for M-ary OFSK.  $M = 2^N$ , where N is bits per symbol ( not noise ). Bit rate/Bandwidth is maintained at 1.0 for each symbol frequency, while  $W = Mf_b/N$ . C/N becomes  $E_b/\eta$ . **The R effect is = W/R.**

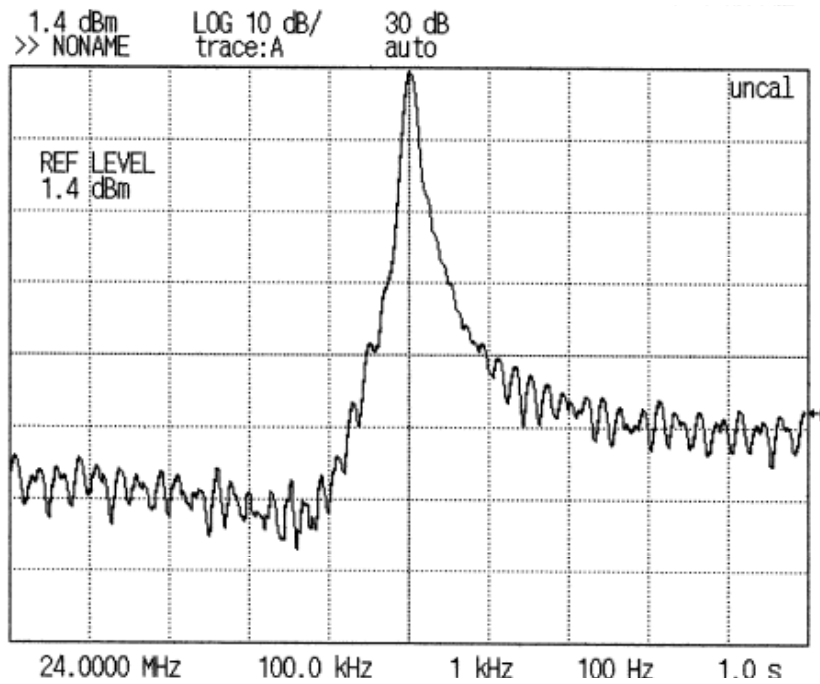
$$P_e = [(M-1)/2] \operatorname{erfc}[N E_b/2\eta]^{1/2} \text{ ( from Taub and Schilling [2] ) .}$$

Compare this to  $P_e = 1/2 \operatorname{erfc} [\operatorname{SNR}]^{1/2}$  which applies when N = 1. Obviously increasing N and M raises the effective value of the SNR.

### References:

- [1] R. E. Best, "Phase Locked Loops", McGraw Hill.  
Eq. 3.82.
- [2] Taub and Schilling, "Principles of Communications Systems", McGraw Hill.  
R effect, Sect 9.5, PLL Sect. 10.7, Narrow Band Noise Sect 7.5, --Correlator pp454. W and Pe formulas pp481 ( Table 11.20-1 )
- [3] T. Rappaport, " *Wireless Communications*", Prentice Hall.
- [4] K. Feher, "*Telecommunications Measurements, Analysis and Instrumentation*", Noble Press.
- [5] J.C. Bellamy, " *Digital Telephony*", John Wiley.
- [6] K. Feher, " *Wireless Digital Communications*", Prentice Hall.
- [7] Carlson, A. B., " *Communications Systems*", McGraw Hill. ( On OFSK )
- [8] Sklar, B., " *Digital Communications*", Prentice Hall ( On OFSK ).

## Appendix.



A1. Swept Response of a 3 stage ultra narrow bandpass filter ( Fig. 5 ) with near zero group delay at a single frequency. As can be seen, the amplitude noise bandwidth is extremely narrow, but the Nyquist bandwidth is very broad.

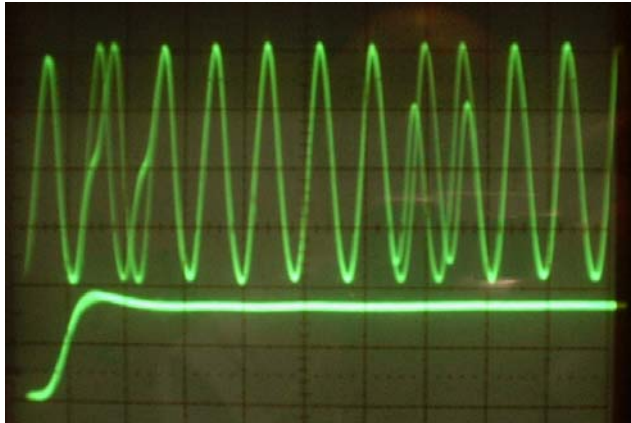
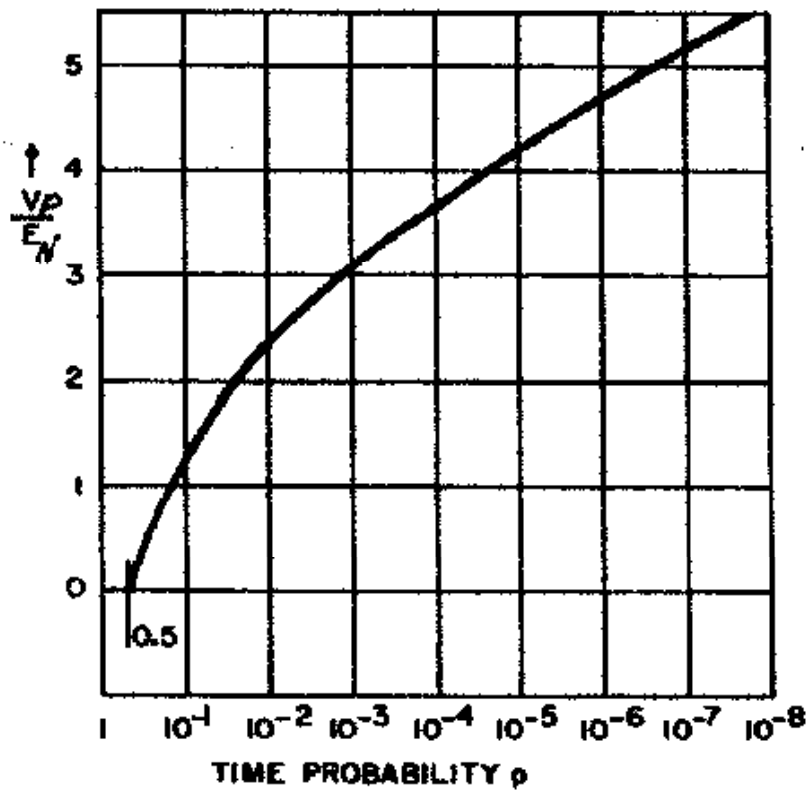


Fig. A2. 90 degree phase modulation for 7 cycles with NRZMSB. This is a 7 cycle bit period.  
 $\tau = 7$ .



The Q(z) curve.  $V_p$  is the peak signal amplitude.  $E_N$  is the RMS noise level. The time Probability is the error rate. The BER for 3PRK follows this curve. Increasing  $\tau$  will result in a dramatic decrease in the time probability ( $P_e$ ). This is an excellent application of the R effect.

Excellent results are obtained in space probes using wideband FM with a low detected frequency. The detected frequency being assumed to be the bit rate.