

Negative Group Delay Tutorial

H. R. Walker

Pegasus Data Systems (7/16/11)

The group delay of a filter is determined mathematically from the following relationships and is easily measured with an RF pulse, or Dirac impulse.

$$T_g^+ = \frac{\Delta\theta_-^+}{2\pi\Delta f_-^+} \quad \text{Eq. 1.}$$

The group delay is positive when the phase shifts from - to + as the frequency rises, which is from lead to lag in phase. **Notice the + and - signs in the above equation.** Positive group delay in a filter is associated with a rise and fall time in level when responding to a pulse. A second relationship is: $T_g^+ = \frac{Q}{IF}$, showing group delay is related to resonance Q. See reconciliation note at end.

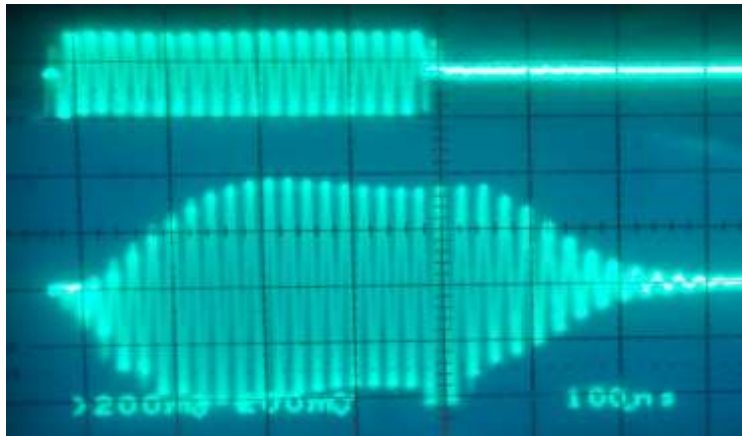


Figure 1. The pulse response of a conventional filter having positive group delay which meets the Nyquist criteria (4) for $BT = 1$. The rise and fall time T is approximately 200 nanoseconds. The bandwidth B of this filter = 5.0 MHz. The Q is approximately 40 at an IF of 48MHz. The upper trace is the input, the lower trace is the output.

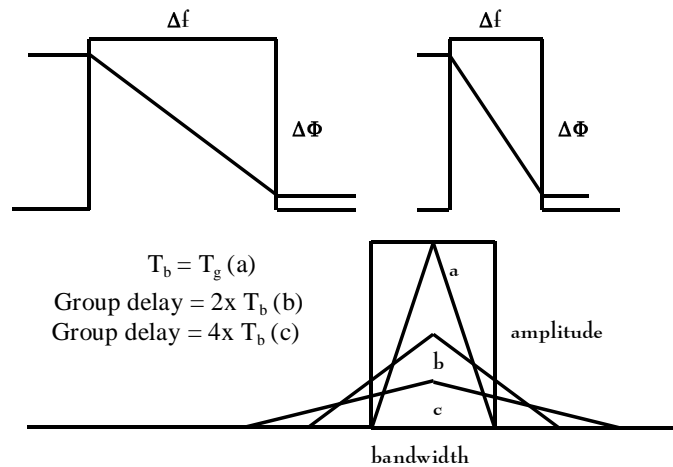


Figure 2.

The ideal, or brick wall filter, is shown in Fig. 2. This filter has a phase shift ' $\Delta\Phi$ ' from edge to edge of 180 degrees, or π radians, over an edge to edge frequency shift " Δf ". Normally, $\Delta\Phi$ and Δf cover one bit period. If the group delay is longer than the bit period, or pulse period, as shown at the bottom, the filter output level decreases to a level = Bit Period/Group delay. (T_b / T_g).

This is the filter proposed by Nyquist, which does not exist in practice, but is approached with the raised cosine filter when $\alpha = 0$. All such filters, including most crystal filters and the familiar LC filters, have group delay, which is equal to the rise time seen in Fig.1.

$$T_g^- = \frac{\Delta\theta_+^-}{2\pi\Delta f_+^+} \quad \text{Eq. 2.}$$

Negative group delay occurs when the phase shifts through the filter from + to - as the frequency rises. Note the change in + and - signs above.

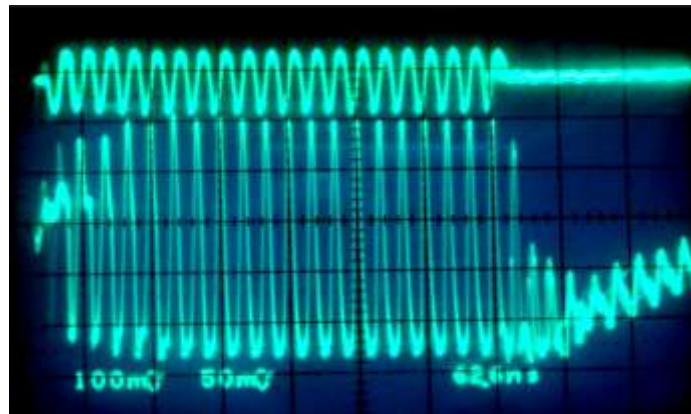


Figure 3. A filter having negative group delay has a group delay (rise time) approximately equal to one cycle of the carrier pulse frequency. There is no slow rise time (integration) as in the conventional filter shown in Fig. 1. Since an event cannot be anticipated, group delay actually can never be less than zero, which occurs if the filter frequency is infinite. The filter effect must be causal.

The Nyquist sampling bandwidth (rate) B is still $1/T = 1/(\text{Pulse rise time}) = 1/(\text{1 cycle period}) = \text{the IF. (4)}$. This bandwidth is much broader than for conventional filters. The Q according to the equation above is near zero. The 3dB noise bandwidth of these filters is much less than for conventional filters, despite the very low Q. See Figures 4 and 11 below.

Figure 4 shows the amplitude response and phase shift of the bridge or half lattice filter, which is one of a group of negative (zero) group delay crystal filters. The same plot applies to the series emitter filter.(Figure 5). Note that the phase shift is from a positive to a negative value at the peak, indicating negative group delay. Outside the filter bandpass the phase shift is constant, indicating no group delay. A maximum positive group delay is indicated at the dip.

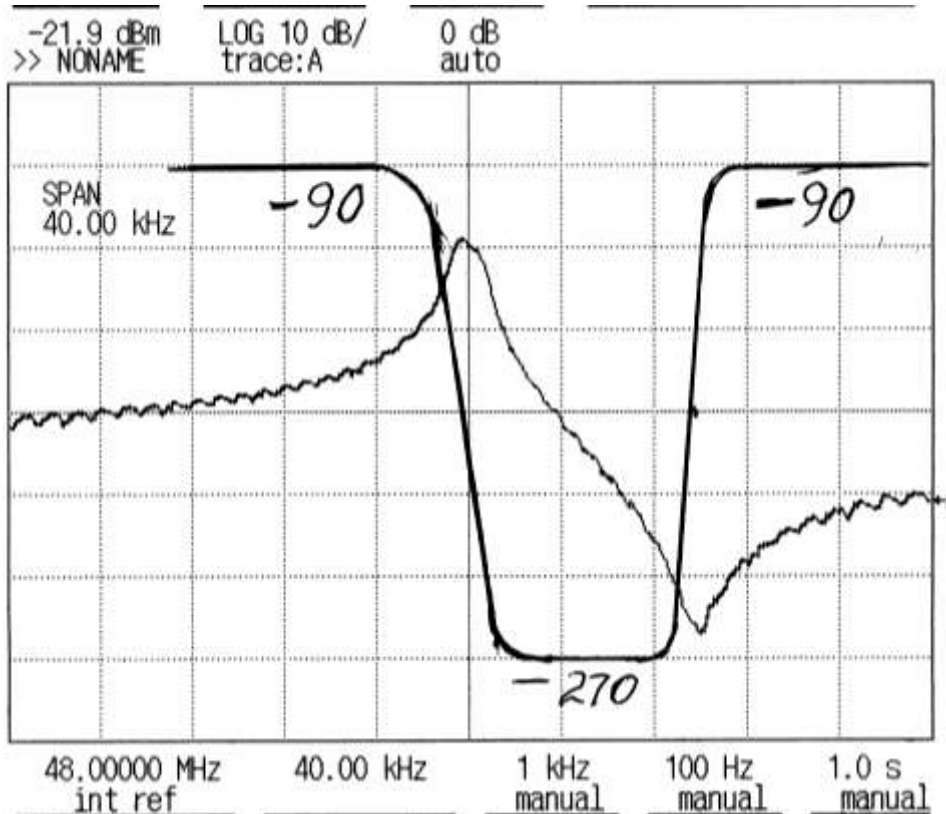


Figure 4. Response and phase shift of the series mode near zero group delay filters. The peak to shoulder response is approximately 20 dB per stage.(10 dB and 4 kHz per scale unit).

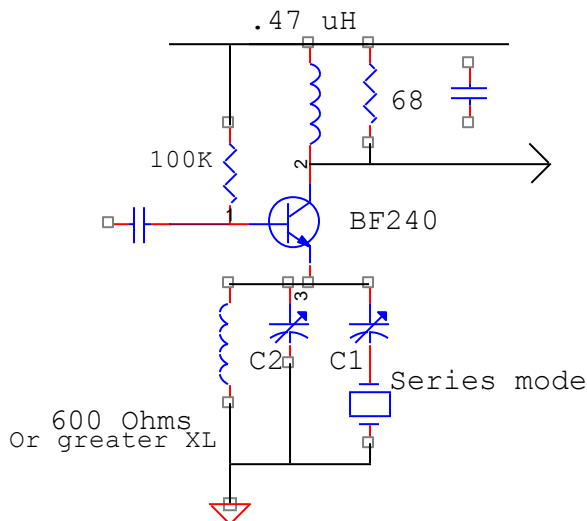


Figure 5. Schematic of the series emitter near zero group delay filter. These stages can be cascaded to reduce the sideband levels, thereby reducing the out of band noise as well to extremely low levels. The filter has excellent dynamic range and very little harmonic distortion if not overdriven. Due to the very narrow 3dB noise bandwidth, this filter will reduce the sidebands in any AM pulse or abrupt phase change modulation system. **The carrier alone retains the necessary pulse information when using negative group delay filters since the components of the Fourier spectrum are separable.** Other filters are

described in Reference (5). Feedback slightly improves the fundamental frequency gain, reduces harmonic response and reduces the residual stored signal level between pulses. All negative, or zero group delay filters, can be cascaded to obtain greater shoulder reduction of any sidebands. See Figure 11.

There are LC filters using feedback that can also exhibit negative group delay. The 'Zero Group Delay Q Multiplier' (ZQM) filter is one known example. See also Figures 11, 12, and 13.

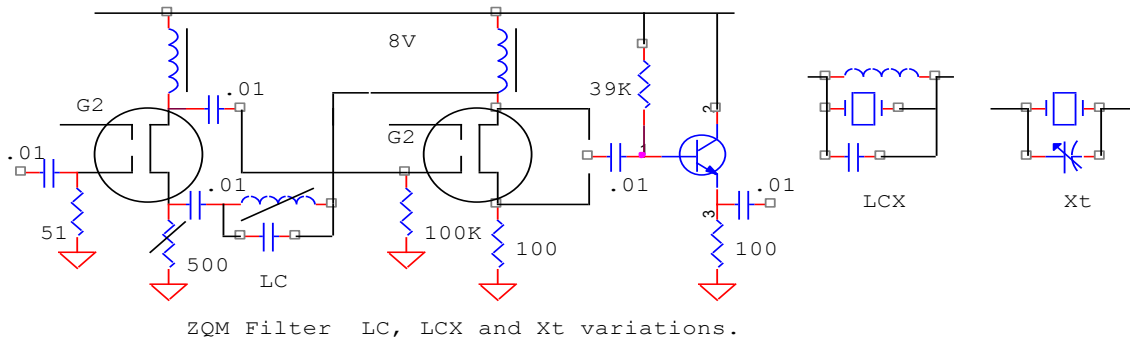


Figure 6. The ZQM filter.using FET devices. Adjustable feedback determines the Q.

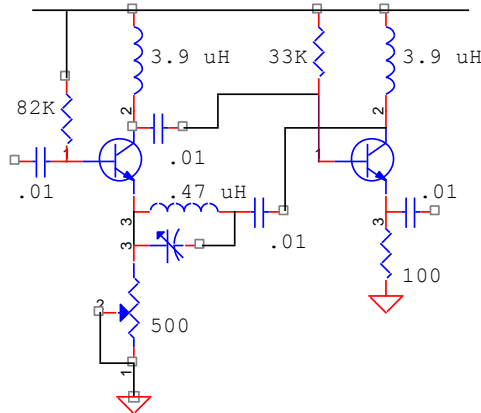


Figure 6a. ZQM filter using RF transistors

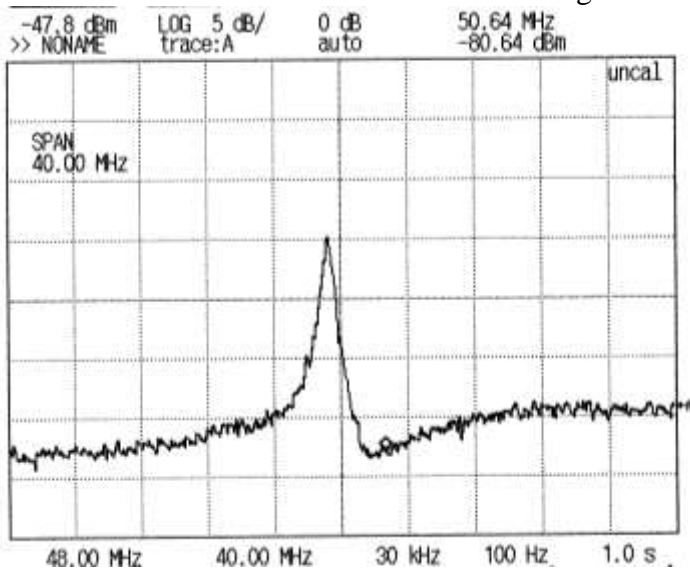


Figure 6b. Bandpass of Fig.6.

Figure 7 shows the measured group delay (blue trace) of the ZQM filter when the Q is approximately 50. The group delay is shown to be negative above and below the peak.

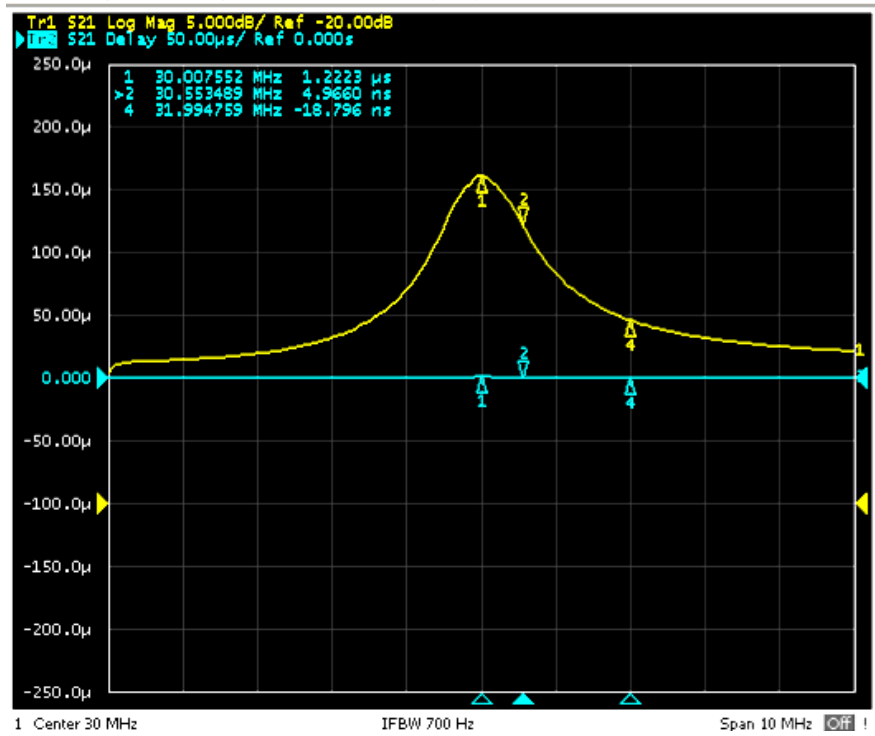


Figure 7. Network Analyzer plot of ZQM filter showing zero group delay (blue). At 32 MHz, the cycle period T is 33 nanoseconds. At the peak there is 1.22microsecond group delay. At the 3 dB point on the response curve the measured delay is 4.9 nanoseconds. Beyond this the group delay becomes negative. This filter could be operated closer to the peak without serious group delay loss. The magnitude is shown in yellow.

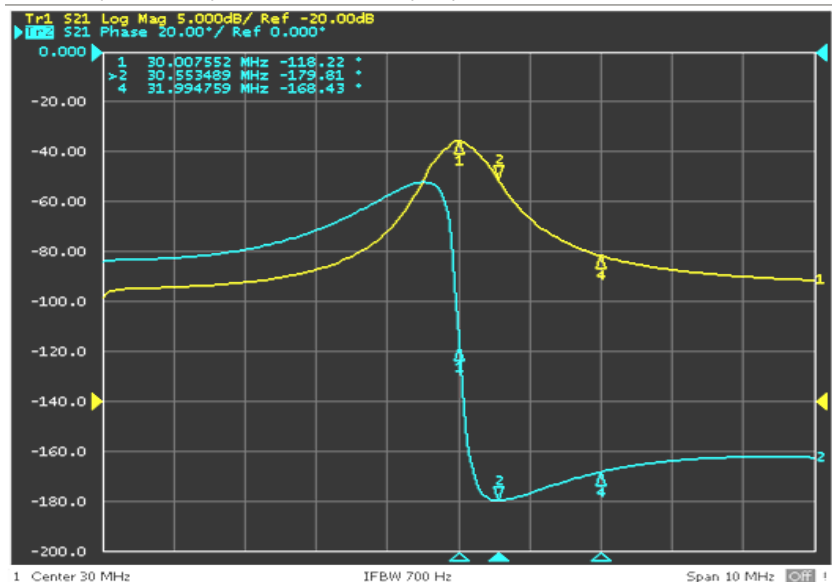


Figure 8. The phase shift of the ZQM filter. The + to - shift indicates negative delay.

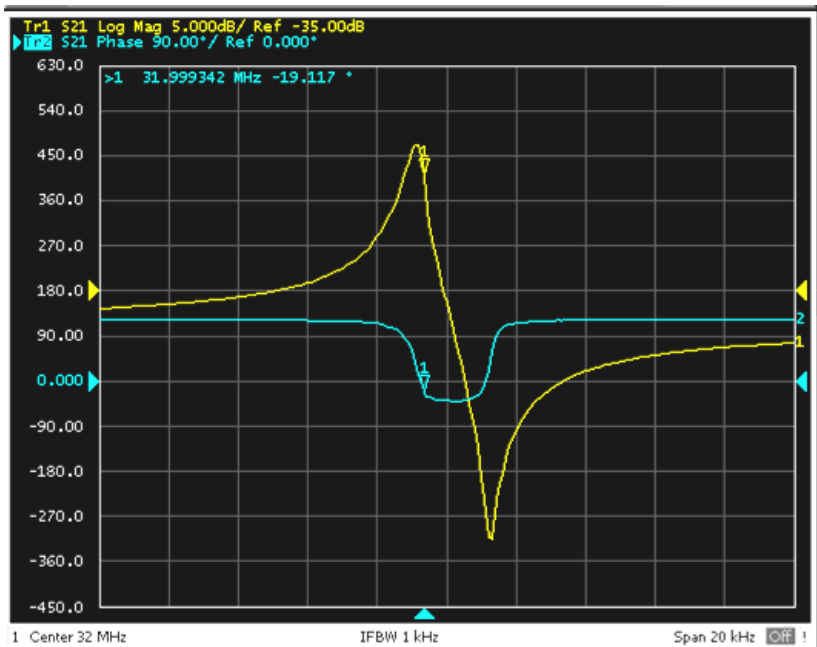


Figure 9. Network Analyzer plot of the series emitter filter (Fig. 5) showing amplitude response as the yellow line and negative phase response as the blue line. Note the similarity to Figure 4.

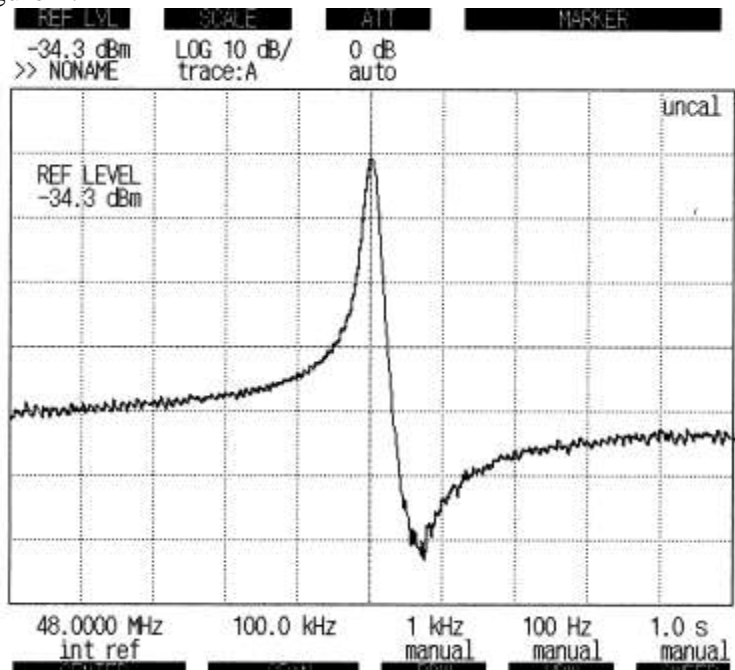


Figure 10. The bandpass response of cascaded UNB zero group delay filters, such as the one in Fig. 5. Any sidebands, or out of band signals more than 10 kHz away from the peak, will be attenuated 40 dB. The filter obviously has a very narrow noise bandwidth. (Approximately 1 kHz at the 3 dB points). However, the $BT = 1$ relationship yields a T of 1 IF cycle and an actual bandwidth = to the IF when the group delay is negative or zero.

A ‘Google’ or ‘Bing’ search for ‘Negative Group Delay’ will show prior work in this field. The most commonly cited example of a negative group delay filter is the operational amplifier version. This is a feedback filter similar to the ZQM concept. **The Network analyzer plot is the same as for the ZQM filter.** A ‘Gyrator’ principle may be involved.

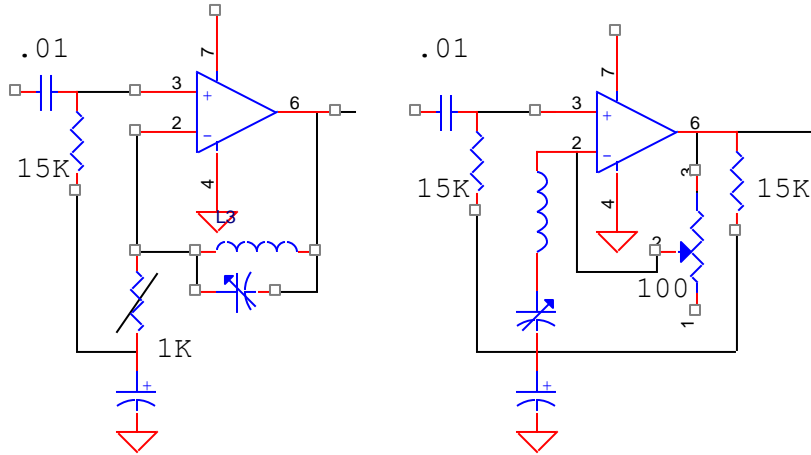


Figure 11. Operational Amplifier realization of a negative group delay filter.

(These circuits have a tendency to oscillate – care is need in application)

This filter can operate as a Q multiplier with a resulting Q much higher than the Q of the LC pair. The feedback resistance determines the Q. A resistor across the LC will influence Q and circuit gain. The filter at the left has not been found to be a good filter for RF use, as it is too temperature unstable and must be operated too far off peak. The circuit on the right is more stable. It is necessary to use an Op Amp with very high frequency roll off in these circuits. A CLC449 was used in testing, but this component is no longer available. A TSH 310 (ST) can be used up to 48 MHz. The circuit must be off tuned to the 3 dB point for zero group delay as seen in Figures 7 and 8.

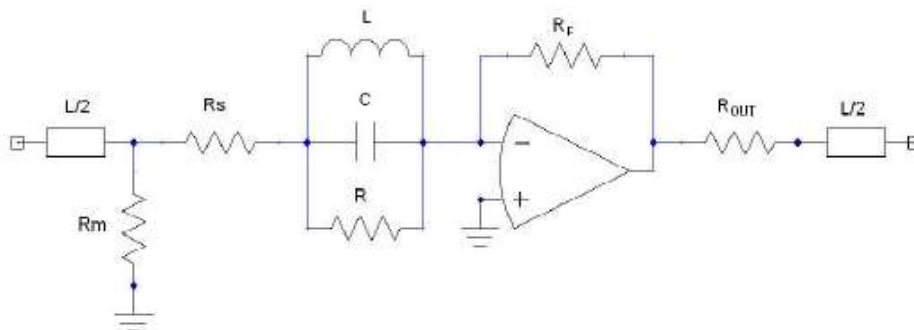


Figure 12. A circuit proposed by Kandic and Bridges at University of Manitoba, Reference (9).

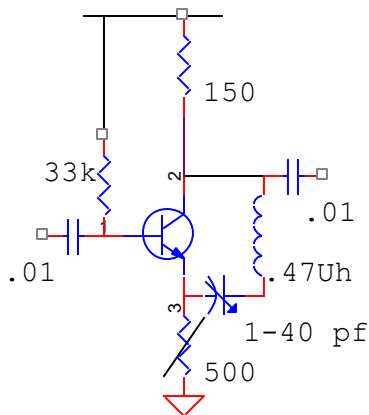


Figure 13. Feedback Circuit.

The feedback circuit in Figure 13 will also exhibit negative or zero group delay, but at a slightly lower Q than the Op Amp and ZQM circuits. Substituting an inductor for the 150 Ohm resistor can result in a circuit with higher Q that may oscillate at extreme settings. Substituting a 500 Ohm pot. for the 150 Ohm resistor enables the circuit to be adjusted for highest Q. A Q of approximately 40 is in practical circuit use. Adjusting the bias may also help to increase Q. This filter is stable and has been used satisfactorily with UNB data modulation. See Q multiplication below.

Note

All of the feedback type filters may have a tendency to oscillate unless strongly driven. They will exhibit negative group delay when the incoming signal is strong enough. They sometimes cannot reliably be used at low signal levels – at a receiver input for example.. *It is the opinion of the author, based on breadboard experience, that OP Amp filters are too unreliable for practical use. The frequency drift is too great at high Q levels and they are level sensitive.*

Crystal filters of the types described here with a 3dB noise bandwidth of 500 Hz to 1 kHz have measured group delays of 250 to 500 microseconds on the network analyzer. *Obviously this does not apply if the group delay is negative.* As an exercise, the group delay of a positive group delay filter can be calculated from the equation $T_s^+ = \frac{Q}{IF}$

Applications:

The zero group delay filters can be used advantageously with any AM pulse or abrupt phase change modulation method. These methods create a Fourier spectrum from which the spectral components are separable if negative group delay filters are used. There must be no FM. *The carrier can be separated and used without sidebands.* This has been demonstrated with Radar type pulses (8) and in “Ultra Narrow Band “digital data modulation systems. These data systems utilize end to end AM pulse width modulation where a digital one is a pulse having a carrier phase 1 and a digital zero pulse has a carrier phase 2. Papers on this method have been published since 1998. (References 5, 6,

7). Because energy can be stored in the zero group delay filter crystal, a vector adding method is used to separate the pulse phases in some methods. The series emitter crystal filter stores very little energy from a pulse A homodyne (synchronous) phase detector is used. The application of negative group delay filters and processing gain has more recently been applied to AM pulse modulation as used in Radar, IFF, DME, Tacan, PWM, PAM and PPM methods, as well as to the various data modulation methods.. (References 8 and 10).

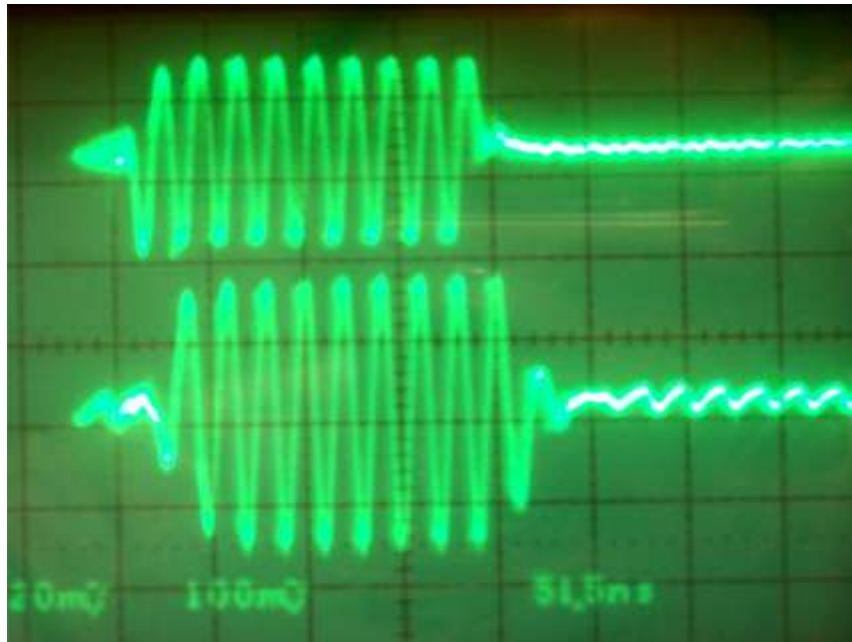


Figure 14. The waveform after 3 stages of the filter shown in Figure 5. The sidebands have been reduced 40 dB. There has been a slight amount of rise time = group delay added, but the pulse comes though clearly with negligible sideband energy. When the pulse rate is low, there is a negligible amount of energy stored in the crystals after the pulse and the pulse is cleanly detected.

The unusual effect is that the Fourier sidebands of the pulse are removed in the process and only the carrier is passed through the zero group delay filter, indicating the **Fourier spectral components are separable**. This leads to the assumption that filtering and signal processing can be used to greatly improve the receiver sensitivity. The separable method **does not work with ordinary AM, or with FM/PM**. It only functions with pulses where there is a Fourier $\sin x/x$ spread in the spectrum and individual frequencies can be isolated.

Processing Gain:

The SNR of any system can be improved if the filter noise bandwidth can be reduced. This is similar to CDMA (spread spectrum) methods where processing gain is obtained by bandwidth reduction.

$$G_p = BW_1/BW_2 = (\text{Rise time 1})/(\text{Rise time 2}) \quad SNR_0 = (G_p)SNR_i \quad \text{Eq. 3.}$$

An AM pulse modulation method could have a required Nyquist bandwidth as represented by the full Fourier spectrum of the pulse (for example = 10 MHz). The typical zero group delay filter has a 3dB noise bandwidth less than 1 kHz. (Fig. 11). The bandwidth reduction, and subsequent processing gain SNR improvement, using the zero group delay UNB filter, is $10,000,000/1,000 = 10,000/1 = 40$ dB, which is a very large signal to noise improvement. In a Radar system this amounts to a considerable range increase, or it enables the detection of much smaller targets.

Previous works by others to reduce the bandwidth have also used Wavelet filters. Zero group delay filters have much narrower bandwidths.

The use of negative group delay filters to reduce the noise bandwidth does not violate Shannon's channel capacity equation because the applicable bandwidth with zero group delay filters is equal to the IF, which is much greater than the bandwidth normally associated with the signal, or the sampling rate of the signal.. $BT = 1$ for positive or negative group delay filters. As the filter rise time T varies, so does the Nyquist bandwidth B . Shannon's channel capacity relationship is related to the sampling rate $1/T$ and not to the noise bandwidth of the filter (4).

Reconciliation Note. The equation $T_g = [\Delta\Phi / (2\pi \Delta f)]$ is the general equation based on $\omega t = \theta$. If $\pi/2$ is used as $\Delta\Phi$ to calculate T_g using Q , the result is $T_g = Q/4IF$. This is the time required for a 90 degree phase shift. For 2π radians it must be multiplied by 4. A network analyzer will measure $T_g = Q/IF$. Group delay is specified for $\Delta\Phi = 2\pi$

The $BT = 1$ relationship is derived from the T_g equation. $(\Delta f)T_g = \Delta\Phi / (2\pi)$. Since group delay is based on a 2π phase shift, $(\Delta f)T_g = 2\pi/2\pi$, which becomes $(\Delta f)T_g = 2\pi/2\pi = 1 = BT$.

Q Multiplication

The Q of an LC circuit can be increased by introducing positive feedback.

The general rule for feedback circuit gain is: Gain $K' = K/(1-K\beta)$
 β is the feedback component. A large negative feedback will reduce the gain to zero. A large positive component will cause the gain to increase and the circuit will eventually reach the oscillation point.

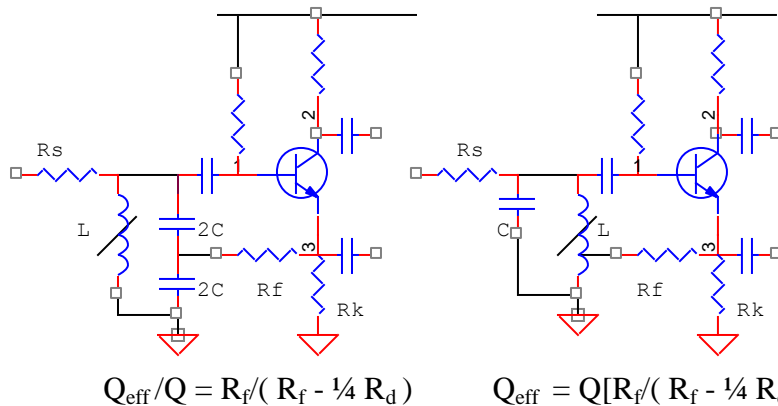


Figure 15.

The circuit becomes unstable and oscillates when $R_f = \frac{1}{4} R_d$
(assuming a center tapped relationship, or that $C = 2 \times 2C$ in series.

The resistor R_d and the bias resistor in the base dampen Q .

Summary:

- 1) Sidebands are not required for AM Pulse modulation methods. They are separable from the rest of the Fourier spectrum and the carrier alone can be used to obtain the data.
- 2) Zero group delay filters are an absolute necessity for 1) to be true.
- 3) The group delay and rise time of a zero group delay filter is 1 IF cycle.
- 4) The effective Q of the zero group delay resonator is near zero. $T_g = Q/\text{IF}$, or $Q = T_g/\text{IF}$.
- 5) The noise bandwidth of the zero group delay crystal filter is less than 1 kHz.
- 6) The Nyquist bandwidth of a zero group delay filter is equal to the intermediate frequency.
- 7) Using zero group delay filters in UNB systems does not violate Shannon's limit because the bandwidth is $= \text{IF}$ and the sampling rate $1/T = \text{IF}$, which is much greater than the data rate.
- 8) There is no violation of Nyquist's bandwidth theorem, since the bandwidth from $\text{BT} = 1$ is very large – much larger than the pulse period, or the bit period in a data system.

References:

- (1) Google or Bing on “Negative Group Delay RF Filters”.” Wavelet RF Filters”.
- (2) Merrill Sokolnik, “Introduction to RADAR Systems”, McGraw Hill. 1962, pp 21.
- (3) Donald G. Fink and Donald Christiansen, “Electronic Engineers Handbook”, McGraw Hill, 1989. Chapter 25. This reference, especially Chapter 25, is highly recommended for information on Doppler and Resolution using different Nyquist criteria filters.
- (4) Mischa Schwartz, “*Information Transmission, Modulation and Noise*”. pp 168. McGraw Hill, 1951. General Pulse Modulation information and $\text{BT} = 1$ explanation.
- (5) H.R. Walker, “Ultra Narrow Band Textbook”, available for free download from www.vmsk.org. 224 pages. Contains information on various ‘Zero Gdel’. Filters.
- (6) H...R. Walker, “Understanding Ultra Narrow Band Modulation”, Microwaves and RF Magazine Dec. 2003.
- (7) Z. Zhang, “Ultra Narrow Band Modulation Salvages Spectrum”, Microwaves and RF Magazine, April 2009.
- (8) H. R. Walker, “Experiments in Pulse Communication with Filtered Sidebands”, High Frequency Electronics Magazine, September 2010, pp 64.
- (9) <http://www.ursi.org/proceedings/procGA08/papers/BP13p4.pdf>
- (10) H.R. Walker, “Sidebands are not Necessary”. Microwaves and RF Magazine August 2011.