Abrupt Phase Change PM Analysis
4/15/07

Abstract:
The following illustrations and equations from Hund [1] and Howe [2] illustrate the difference between FM and PM. **They show how it is possible to have PM without frequency change or useful sidebands, while still containing abrupt change phase modulation in the carrier.**

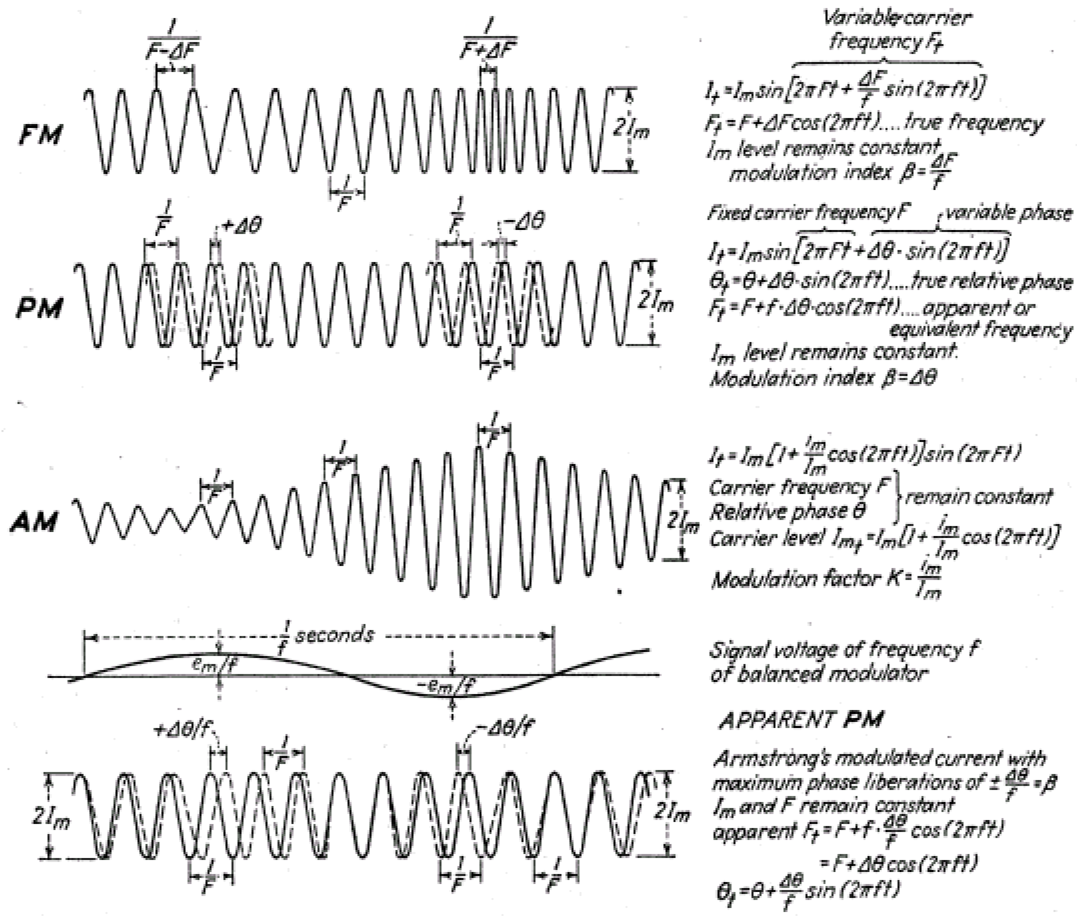

FIG. 18.—Comparison of direct FM, direct PM, AM, and indirect FM.

Fig. A7.1. Differences Between Modulation Methods. Fig. 18, from Hund [1]

With FM the entire component group within the [---] brackets is a variable carrier frequency.

$$I_t = I_m \sin[2\pi Ft + \frac{\Delta f}{f}\sin(2\pi ft)] \quad \text{for FM:} \qquad \text{Eq. 1.0}$$

For PM, applicable to abrupt phase changing PM:

$$I_t = I_m \sin([2\pi Ft] + [\Delta\theta \sin(2\pi ft)]) \qquad \text{Eq 2.0}$$

This equals $F = F_{carrier} + \Delta f$.

With PM, the carrier frequency $I_t = I_m \sin[2\pi Ft]$ remains fixed. There is a **true relative phase** $I_t = [\theta + \Delta\theta \sin(2\pi\, ft)]$**, relative to the un-modulated carrier, but the fixed phase θ in the carrier can be altered at the start by phase switching in the modulator.** If only the variation Δθ is considered, it is $I_t = [\Delta\theta \sin(2\pi\, ft)]$. If Δθ =0, there is no frequency variation.

For the frequency change Δf., this becomes $I_t = [\Delta\theta(+-1)(t)]$ = 0 when the change is abrupt and not a sine wave because Δθ = 0 during most of the period t with a rectangular waveform. See Fig. A7.2 below.

Assume Δθ is a fixed value change and not a variable as with a sine wave. The modulation currents are:

$I_t = I_m \sin[2\pi Ft + -\Delta\theta T]$ if there is no fixed phase θ, or predetermined change in θ. Eq. 3

$I_t = I_m \sin[2\pi Ft + -\theta T]$ When θ is a fixed value shifting according to T. Eq. 4

θT represents a function of θ and t. The actual frequency change is Δf = Δθ/2πt. Eq. 11

Hund's analysis gives the apparent, or equivalent, frequency with change as:

$F_t = F + f\Delta\theta \cos(2\pi\, ft)$, or $F_t = F_{carrier} + \Delta f$. (if Δθ = 0, Δf = 0)..

Eq 2/Eq. 5

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Hund's analysis in the text was limited to sine waves, which create Bessel sidebands.

Let Δθ = zero, then only F remains. Δθ is zero for most of the rectangular waveform period. See Howe's illustration **Fig. A7.2** below. There is no frequency change. **All necessary information is in the carrier F. There will be Fourier sidebands generated that have no effect on the carrier.**

Only when Δθ is not zero, as it would be when it is integrated, RC delayed, or passed through a filter with group delay, is there an apparent change in F_t.

At baseband, the modulating rectangular waveform generates Fourier sideband products. These do no influence the RF carrier, which is at 0 Hz baseband. Cos2πft or sine2πft in equations 2/5 can be replaced by a Fourier series to represent a square wave, but the expansion must be:

$$= \frac{t}{T}\sum_{n=0}^{n=\infty}\frac{\sin n\pi(\frac{t}{T})}{n\pi(\frac{t}{T})} = \frac{t}{T}\sin c n\pi(\frac{t}{T}).$$ Eq. 6

Since the sideband series has no effect on the detected signal, there is no need to use this expansion. The phase change is retained in the carrier according to the above equations and in the Howe [2] equations below. Sidebands are not required as calculated here and as verified in practice. This equation is from Sklar [3], Eq A.24.

Modulation involves superimposing the sidebands on the carrier I_m[(sin(2πC)t +- θ]. Note that the carrier has two states abruptly switched: I_m[(sin(2πC)t + θ and I_m[(sin(2πC)t - θ, Hund [2]. This relationship is also found in Taub and Schilling [4] pp 250, equations 6.1 and 6.2.

$I_t = I_m[(\sin(2\pi C)t +- \theta/2] + 0.5K\{ \sin [2\pi(C+F)t] + \sin[2\pi(C-F)t]\})$ Eq. 7

The two states of the carrier are shown in red.

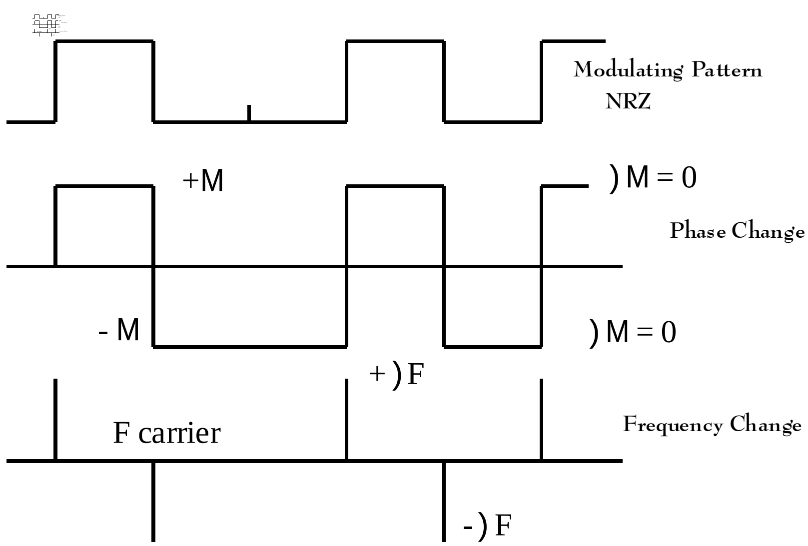
In the present case, $$C \pm \frac{1}{2\pi}\int_0^{2\pi} F(\omega)e^{j\omega t}d\omega,$$ Eq. 8

where the carrier = C and the sideband F is the Fourier integral. The polarity of the sidebands (from $e^{j\omega t}$, which lies within the integral) does not change when they become upper or lower sidebands, as is the case with Bessel functions, or when one of the sidebands is reversed to create a quadrature relationship with the carrier. The vector sum (Fig. A7.3) is the same as that for AM sidebands. VMSK and other UNB methods are variations of BPSK. VMSK uses coded pulse widths, UNB uses phase changes other than +-90 degrees. BPSK* is treated as an AM method.

* Taub and Schilling [4] pp 250

Abrupt phase change modulation of a carrier creates Fourier amplitude sidebands which are not in a quadrature relationship to the carrier. The abrupt phase changed carrier cannot be recreated from the sidebands. Removing the sidebands in a zero group delay filter does not affect the abrupt phase changes in the carrier. The carrier is partially amplitude modulated by the sidebands.

Ultra Narrow Band modulation methods depend upon the unique characteristics of abrupt phase change digital modulation to provide a phase modulated carrier that has no quadrature, or Bessel equivalent, sidebands to cause frequency change. **The method described also has the same phase detected output level when all Fourier sidebands**



are removed.

Figure A7.2. Frequency Change: (According to Howe (2)).

Abrupt phase change digital modulation utilizes a coded baseband with abrupt edges, that is, the rise/fall times are as abrupt, or near zero, as possible. Some RC rise time is inevitable, due to slew rates in the ICs and other parts of the circuitry.

The frequency resulting from a rectangular phase change input is: $F_t = F_{carrier} + \Delta f$. Eq 2/Eq. 9
Δf can be calculated from the basic relationship $\omega t = \Phi = 2\pi ft$. Eq 10
This can be rewritten in derivative form as $\Delta f = \Delta\Phi/2\pi t$. Eq. 11

The rise and fall time t is fixed by the the circuit parameters. During the rise and fall times, there is a large $\Delta\Phi$, which causes a large Δf of very short duration. (about 1 RF cycle). At all other times, $\Delta\Phi$ is zero and the frequency $F = F_{carrier}$. A phase detector using $F_{carrier}$ as a phase reference will detect the phase changes as positive and negative voltages. **The necessary phase changes are retained in the carrier without the need for any sidebands.**

When the variable $I_t = [\Delta\theta \sin(2\pi\, ft)]$, the change in $\Delta\theta$ according to the sine wave causes Bessel products. The lower sideband in a Bessel product series contains phase reversals compared to the upper sideband, which causes a phase and frequency deviation. (Appendix 6).

It is absolutely essential that any bandpass filter used at the transmitter **<u>have zero group delay</u>** to pass the instantaneous change in phase. It will not be broad enough to pass the instantaneous nearly infinite frequency changes. To all intents and purposes, there is no measurable frequency change, but there is a phase change in the carrier that is maintained constant between the rise and fall times. A conventional, or Nyquist pulse shaping filter, has group delay and rise time so that $\Delta\theta$ will no longer be zero.

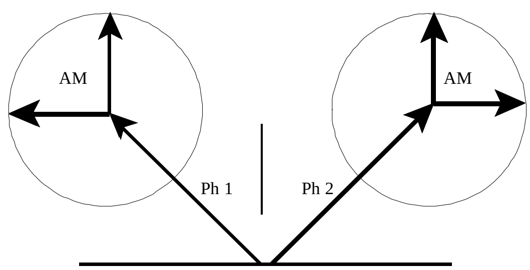


Figure A7.3. Vectors showing the phase modulation vectors Ph1 and Ph2 and the contra-rotating vectors that result from the amplitude equivalent (non quadrature related) modulation portion of the Fourier equations. The Fourier sidebands are not in a quadrature relationship to the phase shifting direction, which is at right angles to the Ph1, Ph2 vectors seen, so that there is no change in the Ph1, Ph2 vector phases.

The Fourier spectrum of the sub-harmonics alone using NRZ-MSB modulation is:
$y(t) = A_{peak} (t/2T_p) [½ +(2/\pi)\cos\pi(t/2T_p) – (2/2\pi)\cos 2\pi(t/2T_p) + (2/3\pi)\cos 3\pi(t/2T_p)$

- $(2/4\pi)\cos 4\pi(t/2T_p) + (2/5\pi)\cos 5\pi(t/2T_p)$ ---]
- which nulls when nt = 1.0. The DC component can be ignored. Eq. 12.
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All harmonics of the bit rate are outside the Nyquist bandwidth. Only the fundamental sub-harmonic remains in the grass spectrum within the Nyquist BW unless T_p is very long. The main thing that changes is the amplitude $A_{av} = A_{peak}(t/T_p)$. In the plots above there is a very large difference in $A_{av.}$

These harmonics and sub-harmonics are of no value in detecting the abrupt phase changes in the carrier. They can be reduced or removed with zero group delay filters.

The full relationship adding the sidebands to the carrier, generated by an abrupt change switching modulator able to accept phase shifts other than +-90 degrees, is given below.

In the case of NRZ-MSB +- 45, or +-60. The carrier form is taken from Rappaport [5], Hund [1], and from Taub and Schilling [4], plus others.

The switched modulation currents for the carrier alone are:
$I_t = I_m \sin[2\pi\, ft]$ for phase one and $I_t = I_m \sin[2\pi\, ft + \theta]$ for phase two. Eq 13.

Adding the Fourier sidebands, we obtain for phase 1:

$$Y(t) = \sin[2\pi\, ft] \pm K[(2/\pi)\cos\pi(T/2T_p) - (2/2\pi)\cos 2\pi(T/2T_p) + (2/3\pi)\cos 3\pi(T/2T_p) - (2/4\pi)\cos 4\pi(T/2T_p) + (2/5\pi)\cos 5\pi(T/2T_p) \text{ ---}] \quad \text{Eq. 14}$$

and for phase 2:

$$y(t) = \sin[2\pi\, ft + \theta] \pm K[(2/\pi)\cos\pi(T/2T_p) - (2/2\pi)\cos 2\pi(T/2T_p) + (2/3\pi)\cos 3\pi(T/2T_p) - (2/4\pi)\cos 4\pi(T/2T_p) + (2/5\pi)\cos 5\pi(T/2T_p) \text{ ---}] \quad \text{Eq. 15}$$

The K is necessary because the relative sideband level shifts with θ and T/T_p.
The sideband level is a maximum when the modulation angle is +-90 degrees and zero when the modulation angle is +- 0 degrees. (sine θ).

The frequency of the carrier is f. The fundamental frequency of the sequential sidebands (also referred to as 'grass') that vary with the data pattern is 1/Tp.

Phases 1 and 2 change in sequence with the data pattern. The sidebands K[---] are unnecessary and are removable in zero group delay filters.

References:

A4(1) August Hund, "Frequency Modulation",. McGraw Hill 1942. This is a very extensive text on FM and PM with extensive mathematical analysis.
A4(2) Howe, Prof., As published in -- K.R. Sturley, " *Frequency Modulation*", Chemical Publishing Co., Brooklyn, N.Y., 1950, Page 9. Figure A7.2 above was published by Prof. Howe in "Wireless Engineer", Nov. 1939. pp 547.
A4(3) B. Sklar, "Digital Communications", Prentice Hall, 2001
A4(4) Taub and Schilling, "Principles of Communications Systems", McGraw Hill.
A4(5) T. Rappaport, "Wireless Communications", Prentice Hall, 1996.