3PSK Modulation

Reviewed 02/02/05

Pulse Position Phase Shift Keying (3PSK) is a Minimum Sideband (MSB) method designed to provide a minimum of Fourier sideband energy for a given number of filter stages, while allowing the use of multistage filters having some short cascaded rise time (group delay). It is a compromise between sinx/x pulse rejection and filter simplicity. The object is to obtain the lowest level of sinx/x spectrum consistent with the group delay of the available filters. The object is to use as few phase shifted IF cycles as practical. It (or 3PRK) are the preferred methods for Cellular Phone use. (See also NRZMSB).

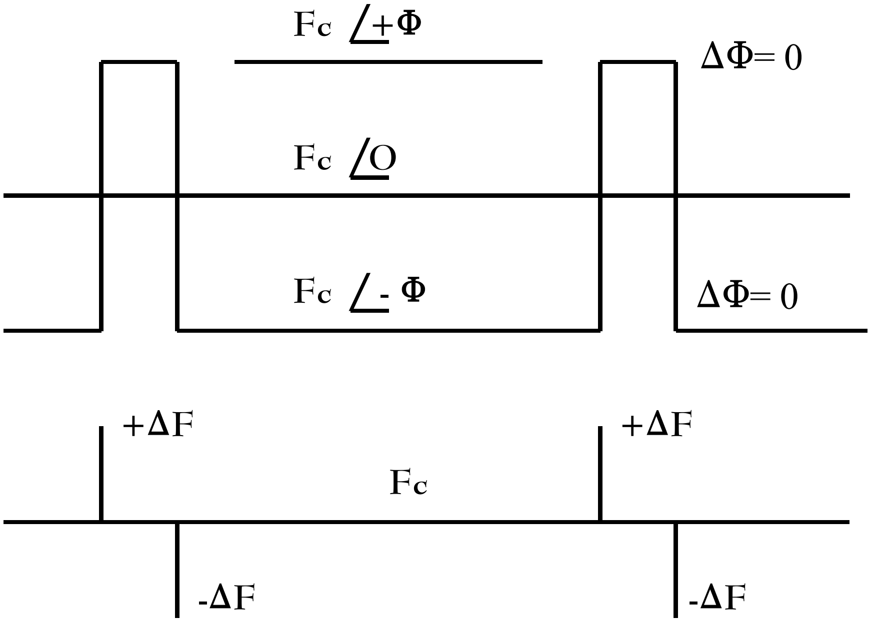
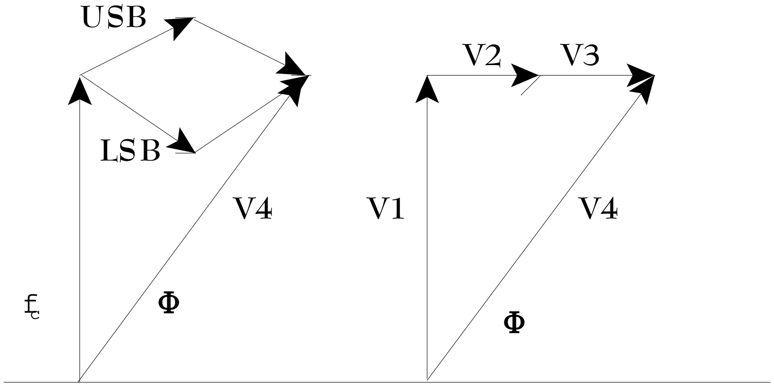


Fig. 1. Abrupt phase change digital modulation utilizes a coded baseband with abrupt edges, that is, the rise/fall times are as abrupt, or near zero, as possible. Some RC rise time is inevitable, due to slew rates in the ICs and other parts of the circuitry.

The frequency resulting from a rectangular input is: $F = F_{carrier} + \Delta f$.
Δf can be calculated from the basic relationship $\omega t = \Phi = 2\pi ft$.
This can be rewritten in derivative form as $\Delta f = \Delta\Phi/2\pi\Delta t$.
The edge rise and fall time is fixed by the the circuit parameters. During the rise and fall times, there is a large $\Delta\Phi$, which causes a large Δf of very short duration. (about 1 RF cycle). At all other times, $\Delta\Phi$ is zero and the frequency $F = F_{carrier}$. A phase detector using $F_{carrier}$ as a phase reference will detect the phase changes as positive and negative voltages.

If a bandpass filter is used, it must have zero group delay to pass the instantaneous change in phase. It will not be broad enough to pass the instantaneous frequency changes at the baseband input edges. To all intents and purposes, there is no measurable frequency change, **but there is a phase change in the carrier that is maintained constant between the rise and fall times.**

Vectors and Bessel Products



Phase Modulation Vectors

Figure 2.

According to accepted practice using PM to generate FM (Armstrong Method), a carrier and two sidebands, an upper and a lower, are required. The vectors for the upper and lower sidebands counter rotate, reaching a maximum in either direction when they are of the same phase. The upper sideband is a signal higher in frequency than the carrier by an amount equal to the modulation frequency. The lower sideband is lower in frequency by the same amount. This gives rise to Bessel sideband products which are necessary to cause the vector V4 to shift in phase. There are three or more different sideband frequencies involved to produce the phase shift Φ. These three frequencies result in spectral spread.

When using abrupt phase modulation, the equivalents of the USB and LSB are seen as V2 and V3. They must maintain the phase shift Φat a constant angle, hence they cannot rotate, but can only reverse. If they do not rotate, they are not at different frequencies, but are **at the same frequency as the carrier V1**.

Abrupt phase angle modulation does not require any frequencies other than that of the carrier**. There are no Bessel products or other frequencies required to produce the phase shift.**

Fourier Products:

The rectangular <u>baseband</u> signal used to produce the modulation has a waveshape that can be analyzed by means of the Fourier transform. If the duty cycle, that is the Φ1/Φ2 ratio is 10%, the modulation result using this baseband code is seen in Fig. 3.

This is the pattern that applies for 3PSK modulation using an abrupt phase change modulator. It is the Fourier spectrum of an <u>amplitude</u> waveform having a duty cycle period that is ON 9/10 of the time, OFF 1/10. Fourier products are amplitude products.

This is an amplitude modulation spectrum, except that the carrier has phase modulation which is not seen.

NOTE: Taub and Schilling explain the absence of Bessel products and the presence of Fourier amplitude products---" The power spectral density and the correlation function of a waveform are a Fourier transform series pair". Result- sinx/x spectrum.

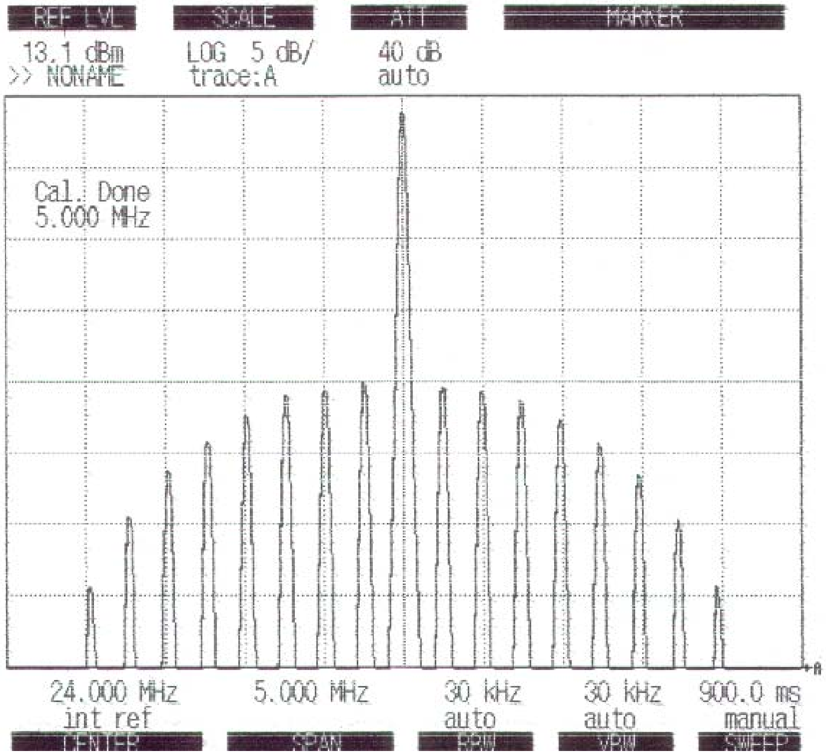
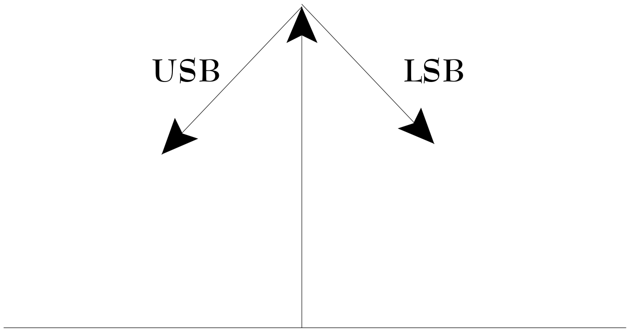


Figure 3. Phase Modulated Spectrum of 3PSK with 10/1 Time Ratio. The scale is 5 dB per div. vertical. The sinx/x Fourier frequency spikes have a peak energy level of -20 dB relative to the data bearing central frequency. The RMS level is -40 dB. Reducing the phase shift period from 1/10 to a smaller ratio will reduce them further. This is without filtering. 30 dB of narrow bandpass filtering will reduce these spikes to -100 dB RMS. This is more than enough to meet FCC regulations under most Parts. (22, 74, 90,95,97).



Amplitude Modulation

Fig. 4. AM modulation Vectors. The sidebands do not change the phase of the carrier. The Fourier frequency spread products are upper and lower AM sideband vectors (Fig 4), which counter rotate to produce a sum which varies the amplitude of the central carrier arrow. **They do not produce any PM. The carrier vector is constant in phase.** Although phase modulation is being used, the Fourier products still appear. They do not produce any PM, but they can have an AM effect on the signal level. A true phase detector with limiter will not see this AM change. The level can be influenced by circuit non linearity.

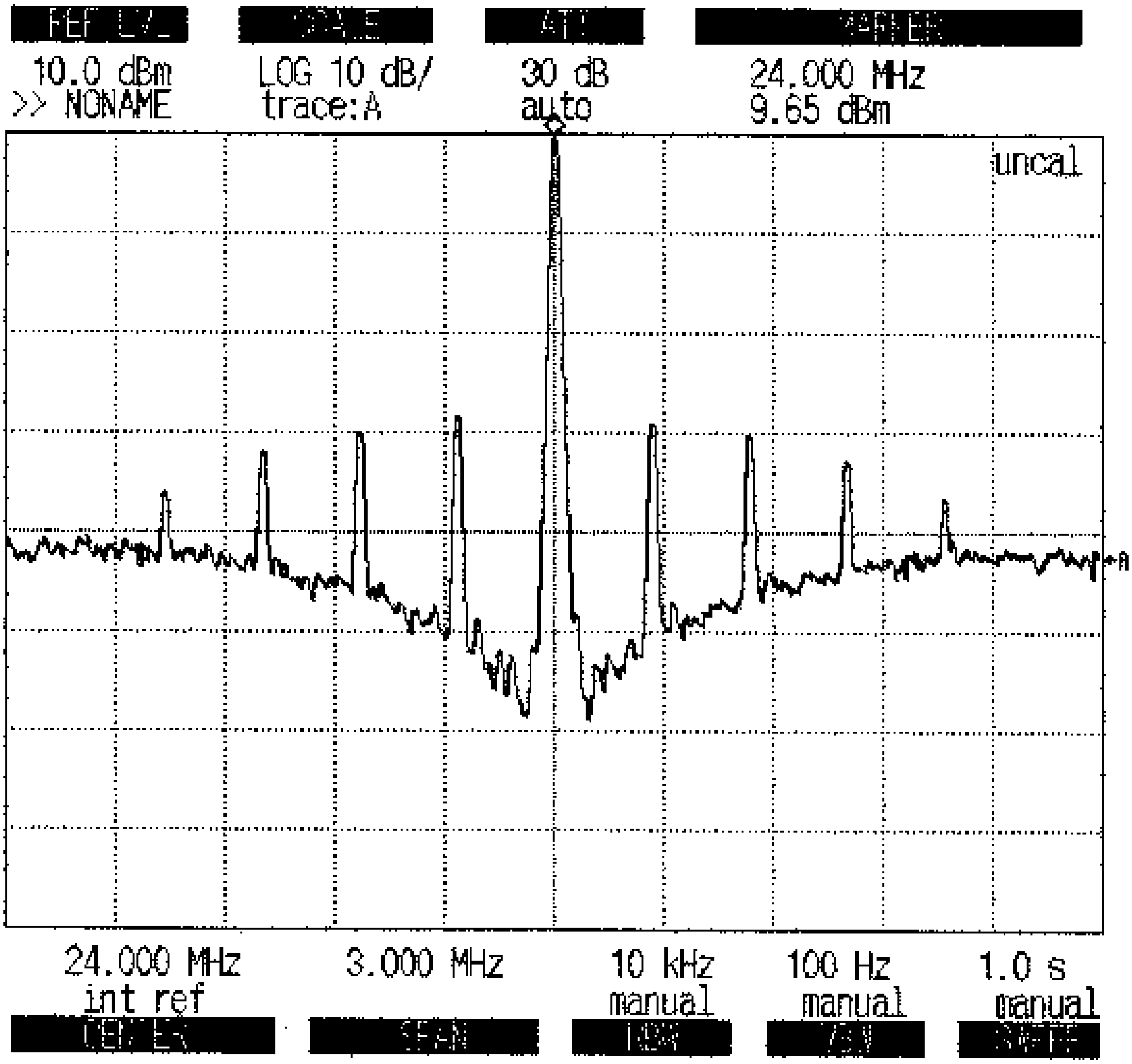


Figure 5 shows the random data spectrum before filtering of a 1/30 Φ1/Φ2 ratio. The lower spikes are sinx/x products resulting from random data at 270 kb/s. These spikes are at -30 dB peak relative to the main energy carrying spike, which is carrying PM with no Bessel or equivalent frequency components - nothing other than the phase shifting carrier frequency. The sinx/x spikes may be considered AM noise. They have a duration of 1/30 bit period during the ON time of the short phase pulse period. The sampling time in the phase detector is much longer.

When both ones and zeros are transmitted using pulse position modulation, thesinx/x spike pattern changes, having fewer sinx/x spikes. It is not necessary to transmit ones and zeros. The decoder is interested only in the ones. The absence of a phase change is decoded as a zero.

The FCC is interested in these spikes and regulates their maximum RMS level. The peak level in Fig. 5 is -30 dB. The RMS level is -60 dB, which allows this pattern to be used on a Cellular system with almost no further filtering at the transmitter. When ones only modulation is used, the RMS level is lower by an additional 6 dB. These spikes can be reduced further by filtering without having any effect on the detected phase output level. Figure 6 below shows the peak and RMS relationship.

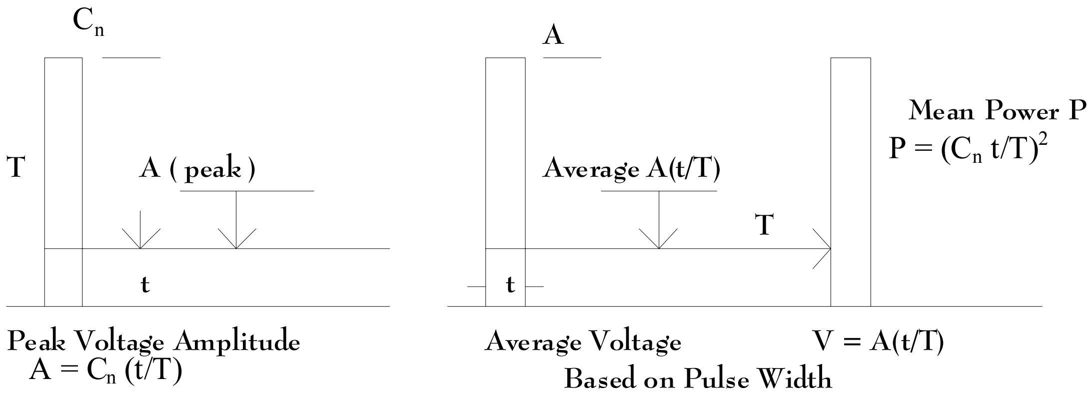


Figure 6. The relationship between peak and RMS levels in the sinx/x spikes. The spikes have a duration ' t ' equal to the shortest phase shift period. The peak sinx/x level relative to the carrier is $-20\text{Log}_{10}(T/t)$ dB. The RMS level is $-40\text{Log}_{10}(T/t)$ dB.

Figure 7.

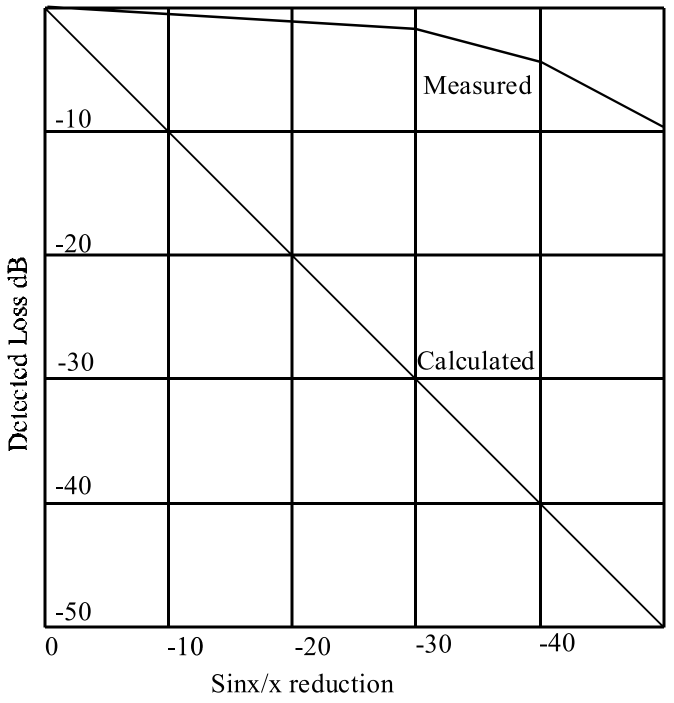


Figure 7. When Bessel products are present, the phase angle is related to the level of the Bessel sidebands by the relationship $2J_1 = \sin \Phi$ [10]. If this relationship were to hold true for the Fourier products, the phase detected output would decrease linearly with the decrease in Fourier sinx/x product level. **Using this formula to calculate the assumed phase angle decrease, and comparing this with the actual detected output level, it is seen that <u>The detected phase angle has almost no change with the decrease in level.</u>**

Figure 7 shows the calculated loss if Fourier products were to cause a loss in phase angle, vs measured detector output when the sinx/x products are reduced by the indicated amount. Without the 30 dB shoulder reduction, the detected peak to peak value was 2.5 Volts. There is only a 2 dB loss when the sinx/x spikes are reduced 30 dB. Adding several cascaded stages of zero group delay filtering does cause some loss, although optimum tuning will yield a higher output level than that plotted in Fig. 7..

Fourier products have no relationship to the detected phase angle.

Figure 8 shows the swept frequency response of a 3 stage zero group delay filter. This filter was used to reduce the Fourier sinx/x spikes for the plot in Fig. 7. All frequencies +- 20 kHz are effectively removed by this filter (reduced by 50 dB). This filter has a very narrow AWGN bandwidth, so very little white noise from the outside world can pass to the limiter that follows.

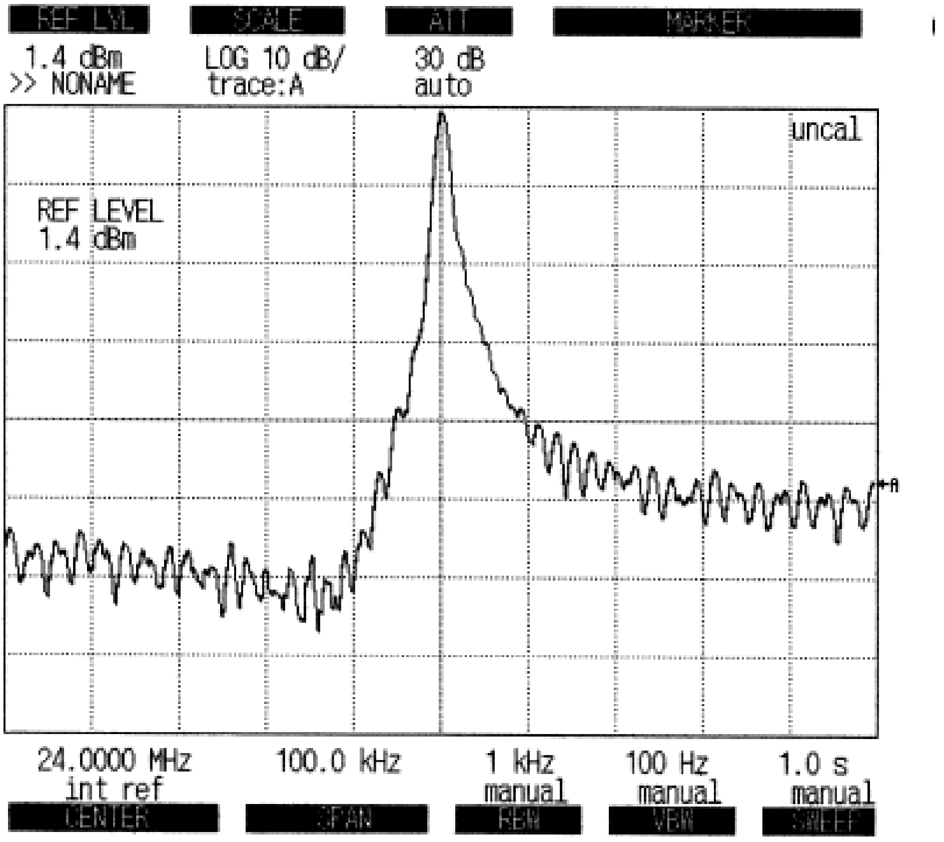


Figure 8. Swept filter response for 3 stages of zero group delay filter.

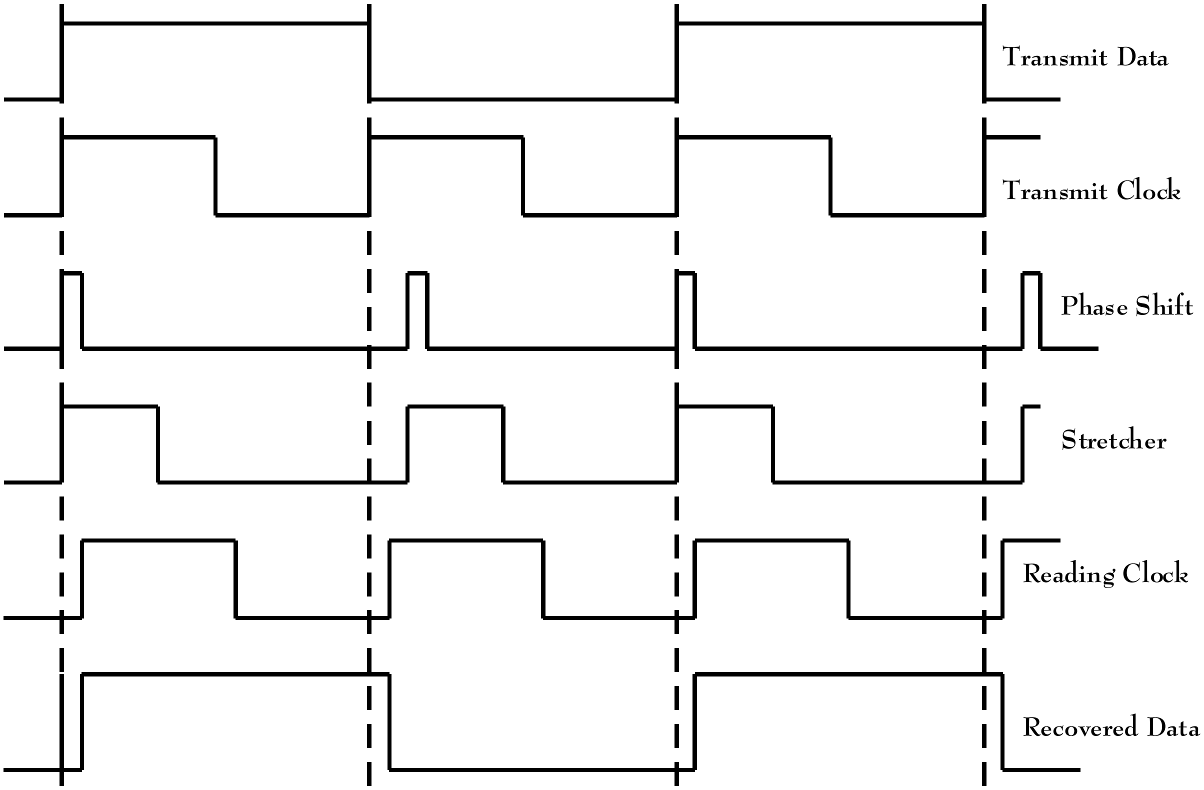


Figure 9. Timing to Encode and Decode the 3PSK Pulses. The detector circuit is given in Fig. 17.

The data and associated clock are shown in Fig. 9. When the data is a digital one, a very narrow pulse 1-3 IF cycles wide is phase shifted at the transmit clock edge. A digital zero has the pulse delayed. As the pulse is detected in the receiver, a pulse stretcher is activated to create a steady level for the data detector. The status of the pulse stretcher is sampled with a properly phased clock to read the data. When the state of the pulse stretcher is high, a digital one is clocked out. When it is low, a digital zero is clocked out.

It is not necessary to transmit a phase shift pulse for a zero, since the decoder does not use it. There is a 6dB reduction in sinx/x spike RMS power when zeros are omitted.

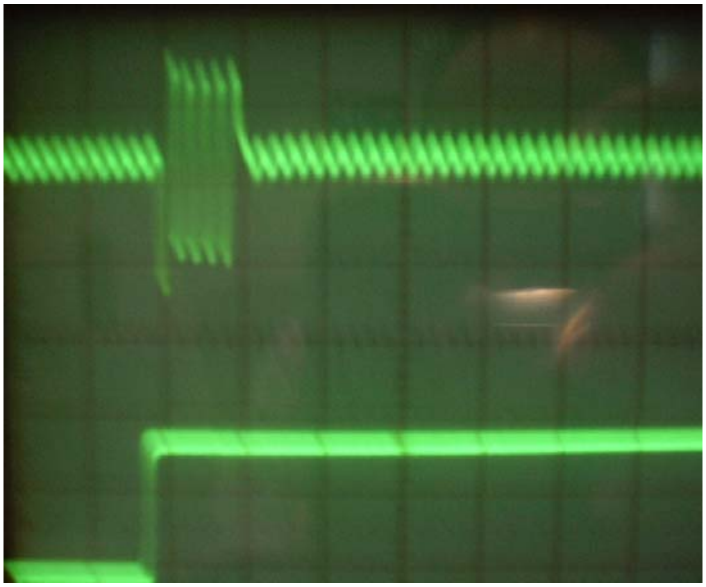


Figure 10.

Figure 10 shows the burst response (output) for a near perfect zero group delay filter. Not all filters have a rise time this perfect. Figure 11 shows the rise time of a shunt filter that is tuned slightly off frequency.

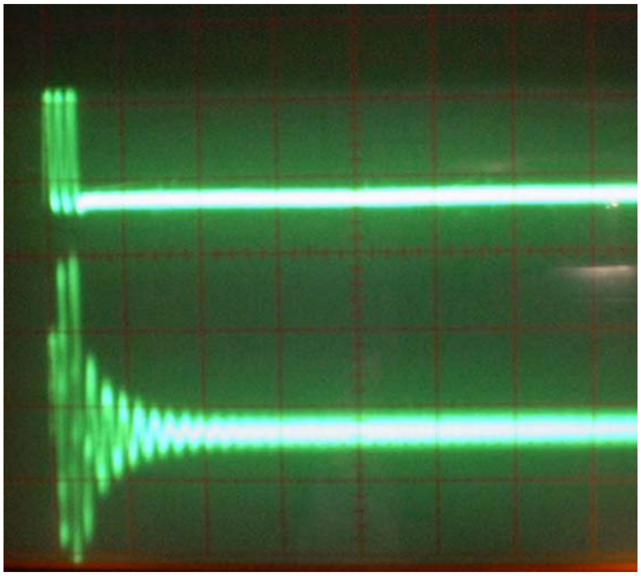


Figure 11. Burst response of a shunt filter with 2-3 cycle rise time. Input top, output bottom. This is the response of the same filter as in Fig. 10, except the filter is off tuned.

The zero group delay filters of figures 10 and 11 have very broad shoulders that allow too much noise and adjacent channel interference to pass in a normal environment. For this reason, a pre filter is used. This is common superheterodyne receiver practice, where there is a RF bandpass filter ahead of the mixer, and the IF filter is narrow band.

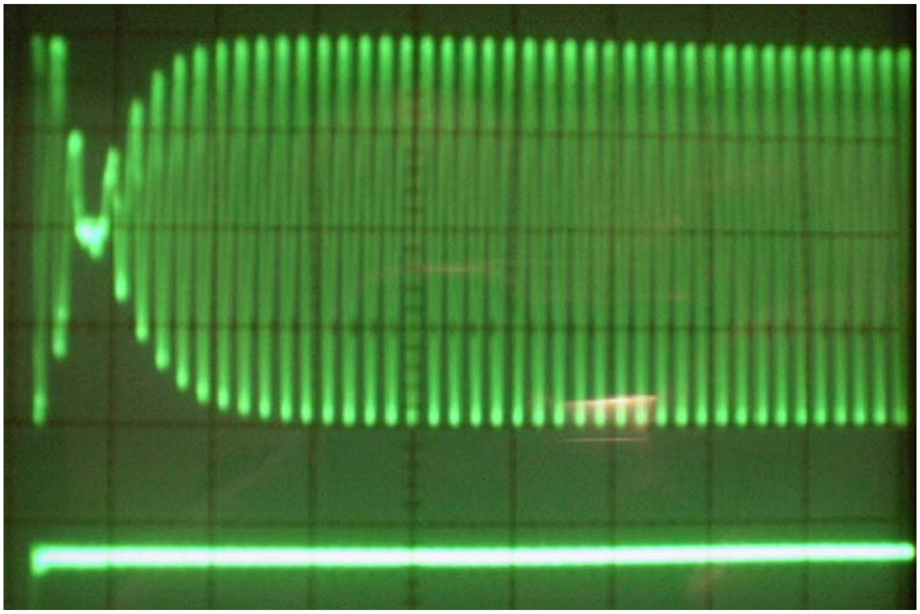


Figure 12. Burst response output of one type of LC pre-filter. Note that the rise time now extends over 4-5 cycles. The preferred pre- filter circuits have better rise time response than this.

Filters, which are cascaded, can have an accumulated rise time that affects the detected signal. For this reason a compromise must be found between the phase shift cycle blocks used and the filters. The maximum data rate is determined from this compromise.

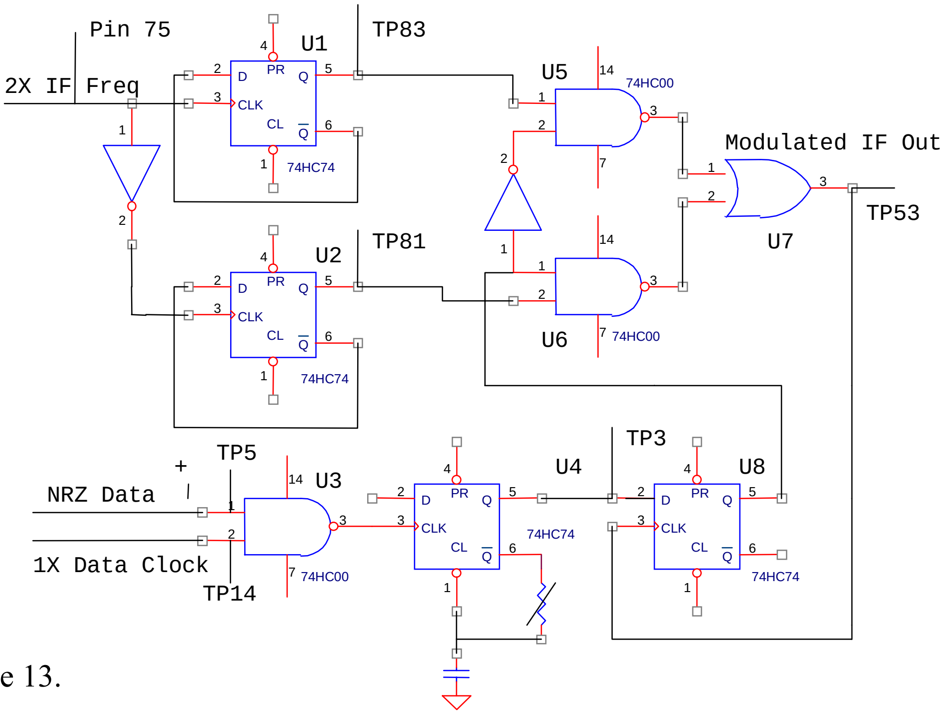


Figure 13.

Figure 13 shows a simple 3PSK modulator for ones only. The first flip flop (top) creates a narrow pulse. The second D flip flop is used to synchronize the data pulse with the carrier cycles. The lower two are divide by 2 counters used to create a 90 degree phase shift. The remaining gates select the phase to be transmitted.

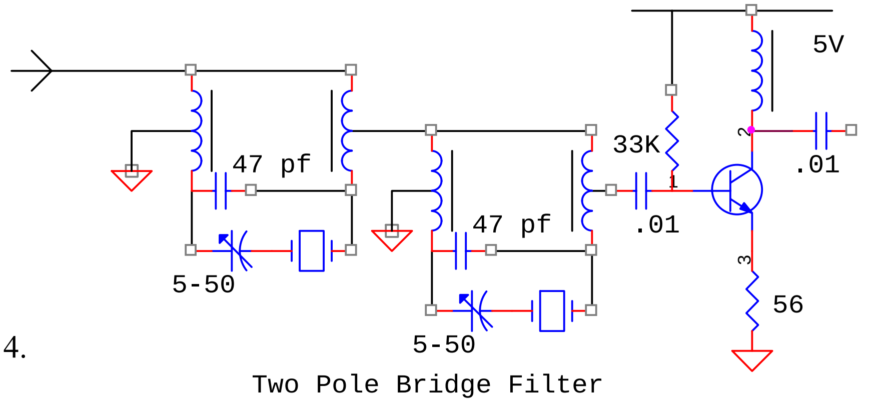


Figure 14.

Figure 14 shows a typical zero group delay filter (Walker Shunt Filter. This is a bridge circuit, which has maximum off balance at the crystal resonance frequency. The effect is that of an RC differentiator, which has no RC time delay. (Response as in Fig. 10). The crystal, which is being tuned to the parallel mode, has a very narrow bandpass. An LC zero group delay pre-filter is used at the input. This filter has a much better group delay response (rise time) than that shown in Fig. 11. Unfortunately it responds to low frequencies, hence the pre-filter is a must..

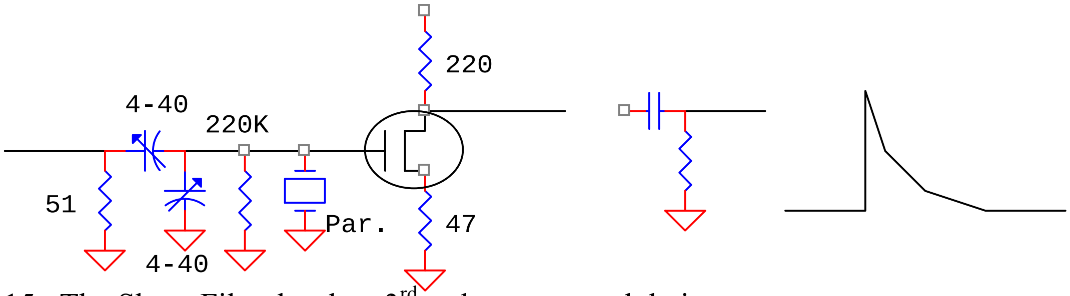


Figure 15. The Shunt Filter has less 3rd order cross modulation.

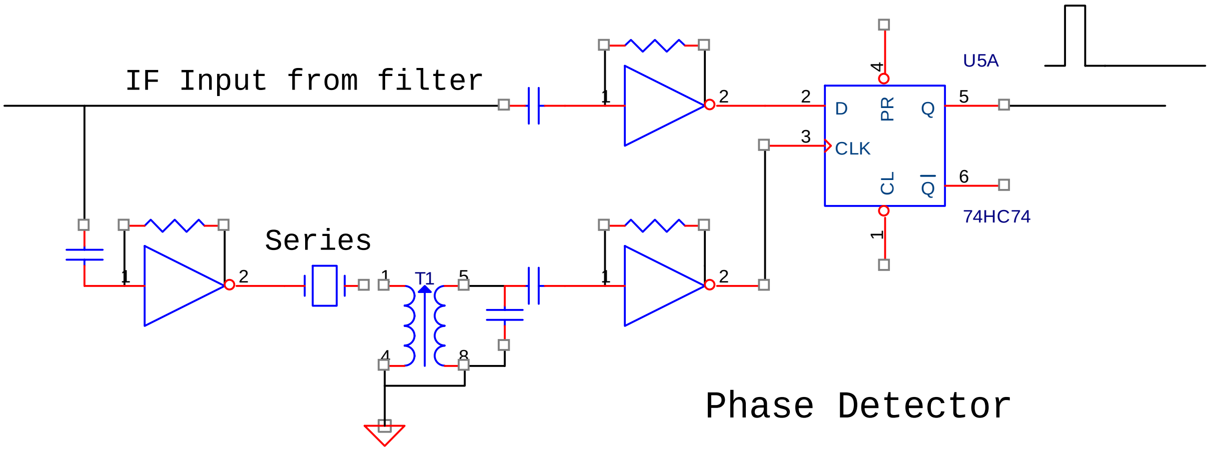


Figure 16. The Phase Detector.

The phase detector has two paths. A limiter drives the circuit at CMOS levels. The upper path carries the modulated carrier. The crystal and transformer derive the phase reference. The phase detector is the D flip flop. This is one of several phase detectors used. An XOR gate can also be used, but it is not as effective in the presence of interference.

Figure 17. Phase detected output for 3PSK when the number of shifted phase cycles is small. If Fc with Φ near zero is used as a reference, only a positive spike will be detected as seen here. Refer to Fig. 1.

This illustration shows a phase shift to indicate a digital one and a delayed phase shift to show a digital zero. In practice only the ones are necessary and only a single spike appears.

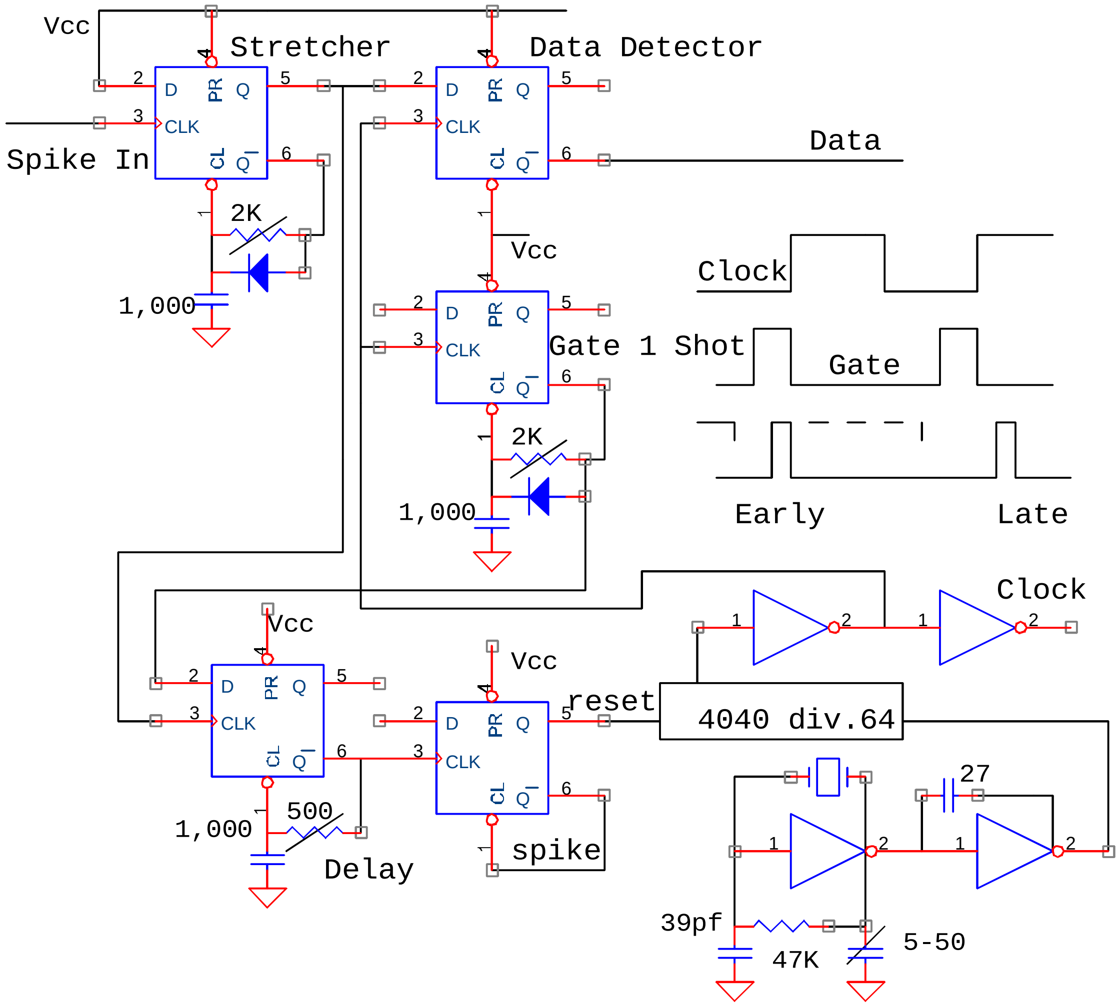


Figure 18. Decoder circuit to restore data and clock.

Figure 18 is the circuit used to restore data and clock using the timing of Fig. 9. The stretcher output is used as the D input in the data decoder to clock out a zero or one. The leading edge also triggers a delay circuit and spike generator to resynchronize the clock oscillator. Digital ones only are used.

The data clock is derived from a crystal oscillator operating at 32 or 64 times the data rate. This is divided down to obtain the clock. During periods when there is no stretcher pulse, the clock free runs.

Taub and Schilling have shown on a basis of error distances that 90 degree phase shifts and 180 degree phase shift have the same P(e). --QPSK vs BPSK.

Shannon's Limit:

$R = W \log_2 (1+C/N)$ or as: $R = (1/\tau) \log_2 (1+C/N)$

It is necessary to understand the meaning of W. It is not the filter noise bandwidth used, but the Nyquist bandwidth, which is and must be equal to the sampling rate. One cannot violate the Nyquist sampling theorem. The general practice is to use 1/(filter rise time) as the Nyquist bandwidth = **(**1/τ**)**. Quoting Schwartz:

"The system channel capacity 'R' is obtained by multiplying the number of samples per second by the information per sample". (Schwartz, [1] pp324 and equation 6-134). The sampling rate is the intermediate frequency, or SuperBit rate. Thus W is = to the Intermediate Frequency.

It is also obvious from the hardware (and Fig. 18 below) that a data rate equal to the Intermediate Frequency can be received and decoded. The actual data rate used is lower, since multiple SuperBits are used for one data bit. Assume a 48 MHz IF, then

48MHz = 48MHz Log_2 (1+C/N)
The equation will balance when C/N = 1 = 0dB. (Shannon's Limit)
Using a lower R, it appears C/N could be below 0dB as in OFSK.

Error Probability:

The error probability for any two phase (2 level) method can be calculated from:
P_e = ½ erfc $[SNR]^{½}$ (Sq. root of SNR is used to get voltage ratio)
P_e = ½ erfc $[(E_s/\eta)\tau]^{½}$ (energy ratio)
P_e = ½ erfc $[(E_b/\eta)]^{½}$
P_e = ½ erfc [z] where z = $V_p/1.4E_N$ (V_p = peak signal level and **E_N** = noise RMS**)**

Note the second of these equations.[$\tau E_{signal} /\eta$], so $\tau E_{signal} = E_b$. = signal power ON for one SuperBit period using 3 PRK. This is true energy per SuperBit. One SuperBit is being detected.

Utilizing a correlator in a post detection circuit, or in the detection circuit, the E_b can be increased by in creasing **τ.**

$$P_e = Q\left[\frac{A}{\sqrt{Nt}}\right] = Q\left[\frac{A}{\sigma}\right] = Q\left[\frac{Cvolts}{Nvolts}\right] = \frac{1}{2}erfc\left[\frac{A}{\sigma}\right]$$

From Feher [2], where A is the peak signal value at the sampling instant and σ is the RMS voltage of the noise power at the threshold detector input. When using a cycle to cycle comparison, noise peak voltage must be compared to signal peak voltage.

$$Q_{(z)} = \frac{1}{2}erfc\left(\frac{z}{\sqrt{2}}\right) = \frac{1}{2}erfc\left(\frac{V}{\sqrt{2N}}\right) == \frac{1}{2}erfc\left(\frac{E_s}{\sigma}\tau\right)^{\frac{1}{2}}$$

Bellamy, Eq C.19, Rappaport D.11

This equation assumes a peak signal voltage V and an RMS noise voltage, which would have peaks at 1.4 times the RMS level. Using a true RMS meter, the relative peak and RMS volts are the same as measured, so the 1.4 correction is not used.

$$Q = \left(\frac{E_s}{\sigma}\tau\right)^{\frac{1}{2}}$$ determines the BER for MSB. τ = 1 to 3 for 3PRK and MCM.

τ = 1 to 3 for 3PRK and 3PSK, but is equal to the number of IF cycles in the entire bit period for NRZMSB.

The theoretical and measured BER follows the Q(z) curve. (page 16).

Now there is another inconsistency with the theory that Bessel sidebands are necessary. The phase detected output is always the equivalent of the transmitted phase modulation angle (Phase change = +-90 degrees, for example) See Fig. 5 above. **There is no detected level change when the sinx/x (amplitude) sideband spikes are raised or lowered**. When passed through a zero group delay filter that reduces the sinx/x spikes by 30 or 40 dB, the detected output level is almost unchanged.

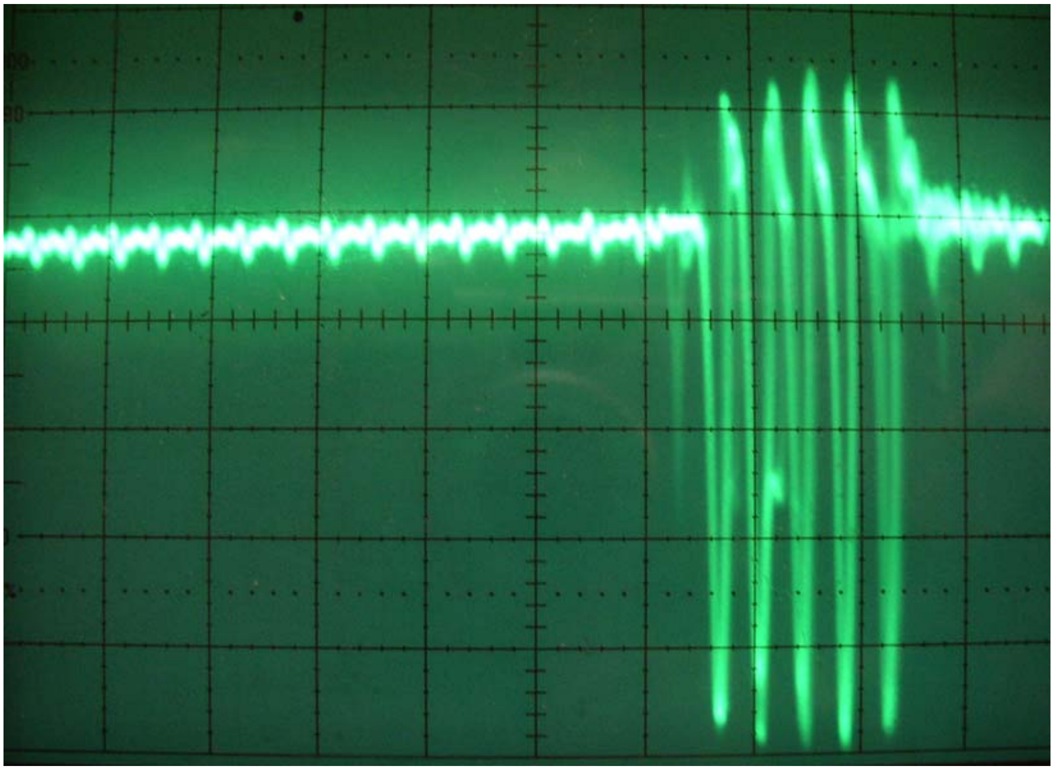


Fig. 19. The detected output of the XOR detector for either the I or Q channel. The filter response time (rise time) is obviously near 1 IF cycle. The sampling rate and Nyquist bandwidth are equal to the Intermediate Frequency.

References:

This is an MSB (Minimum Sideband) modulation method. Other MSB methods are VMSK, WPSK, 3PRK, 3PSK, and UWP. They all use the abrupt phase shift modulator and zero group delay filters. See other papers on this site to cover filters, Shannon's Limit, BER, SNR, etc..

[1] Mischa Schwartz, " *Information Transmission, Modulation and Noise*" McGraw Hill.
Explains pulse position modulation. Has the best explanation of Shannon's Limit.
[2] K. Feher, " *Wireless Digital Communications*", Prentice Hall.
About measuring SNR and BER.

[3] Walker, Koukourlis, Pliatsikas, and Sahalos. " *Wireless Communications Using Spectrally Efficient VMSK/2 Modulation."* In "Third Generation Mobile Telecommunication Systems", Edited by Dr. Peter Stavroulakis. Springer Verlag, Berlin 2001.------Early paper on VMSK.

[4] H. R. Walker, U.S. Pat. 5,930,*303 "Digital Modulation Employing Single Sideband With Suppressed Carrier"*, covers VMSK and VMSK/2. Other patents are pending.
VMSK applied the MSB principle to a sideband.

[5] Taub and Schilling, "*Principles of Communications Systems*" McGraw Hill.
Discussions of Noise and Filtering.

[6] T. Rappaport, " *Wireless Communications*", Prentice Hall.

[7] K. Feher, "*Telecommunications Measurements, Analysis and Instrumentation*", Noble Press.

[8] J.C. Bellamy, " *Digital Telephony*", John Wiley.
Baseband Codes, DC Wander -or Creep- Probability of Error. SNR.

[9] A. Bruce Carson, "Communications Systems", McGraw Hill, 1986.
Good discussion of OFSK.

[10] Hund, August, *"Frequency Modulation*", McGraw Hill 1942.
Best known general reference on PM and FM.

[11] Howe, Prof., As published in -- K.R. Sturley, " *Frequency Modulation*", Chemical Publishing Co., Brooklyn, N.Y.
Spiked nature of abrupt phase modulation was published by Prof. Howe. "Wireless Engineer", Nov. 1939. pp 547.

[12] Best, R.E., *"Phase Locked Loops",* McGraw Hill.
Additional phase noise improvement in SNR is possible. See sect. 3.

[13] Wm. C.Y. Lee, "Lee's Essentials of Wireless Communications", McGraw Hill
To paraphrase Dr. Lee, " The idea is to find ways to slightly mark the carrier wave with the modulation so that the least distortion of the carrier wave is achieved." This is what the MSB methods do.

[14] H.R. Walker, U.S. Pat 6,445,737 " Digital Modulation Device In A System And Method Of Using The Same". Covers 3PRK and MCM, which are MSB methods.
This patent is a practical embodiment of the method described here.

APPENDIX:

90 degree phase shifts as used with 3PSK can be used with an NRZ input that lasts an entire bit period. When used with a correlating detector with integrating filter, the bit error rate is vastly improved since the number of IF cycles (SubBits) are integrated to reduce noise effects.

Remember $SNR = \beta^2 C/N$. **In this equation it is the phase change β that survives the filters that counts. The bit error rate is determined from:**

$$P_e = \frac{1}{2} \operatorname{erfc} [SNR]^{\frac{1}{2}}$$

P_e determines the BER for MSB.

It might appear that β **for 90 modulation is worse than for 180 degree modulation. Actually the phase reference recovery circuit corrects for this and the equivalent β is the same as for 180 degree modulation. (Similar example – BPSK and QPSK have the same BER).**

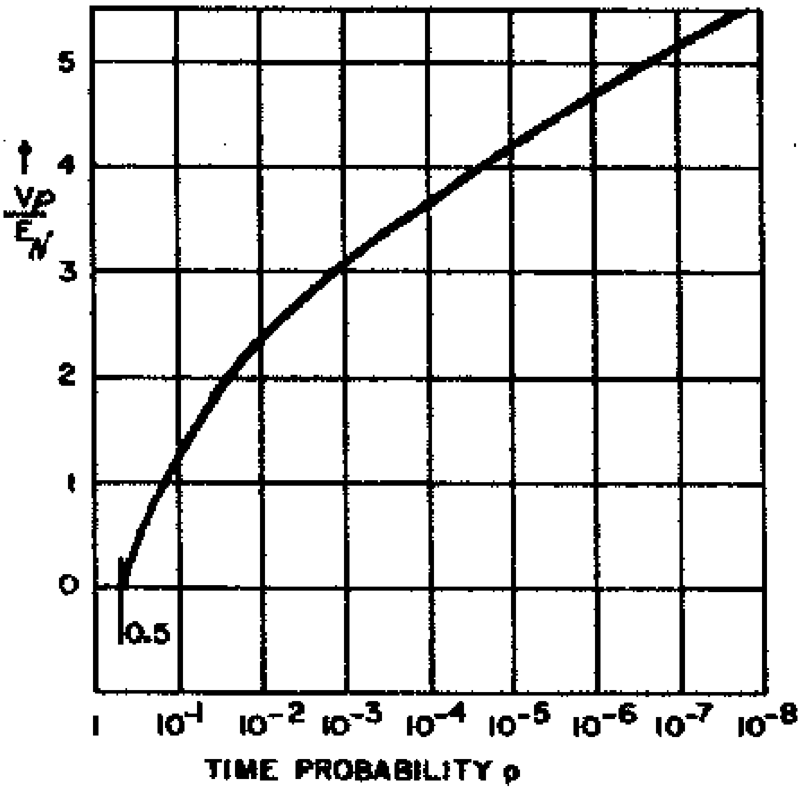


Fig. 1. Amplitude distribution of random noise. V_p is the momentarv amplitude, E_N is the rms amplitude.

Fig. 20. The Q(z) curve. The measured BER for 3PRK and 3PSK follows this curve when $\tau = 1$ or 2.

A correlating detector that integrates a large τ when will have a much better BER. The BER is already 3 dB better than theoretical BPSK when using 3PRK or 3PSK.

However, increasing τ will also raise the sinx/x sidebands, which are a minimum when τ =1.

$A_n = \frac{2At}{T_0}\frac{Sin(n\pi t/T_0)}{(n\pi t/T_0)}$ is the Fourier transform for the spectrum.