

Ultra Narrow Band Modulation on Microwave Links

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Abstract:

Ultra Narrow Band Modulation is a unique phase modulation method that requires no Bessel sidebands. All of the useful phase modulation required for transmission and detection is contained in the carrier, or the J_0 product. This may seem impossible, since it contradicts accepted theory. The explanation however is quite simple and verified from several textbook sources.

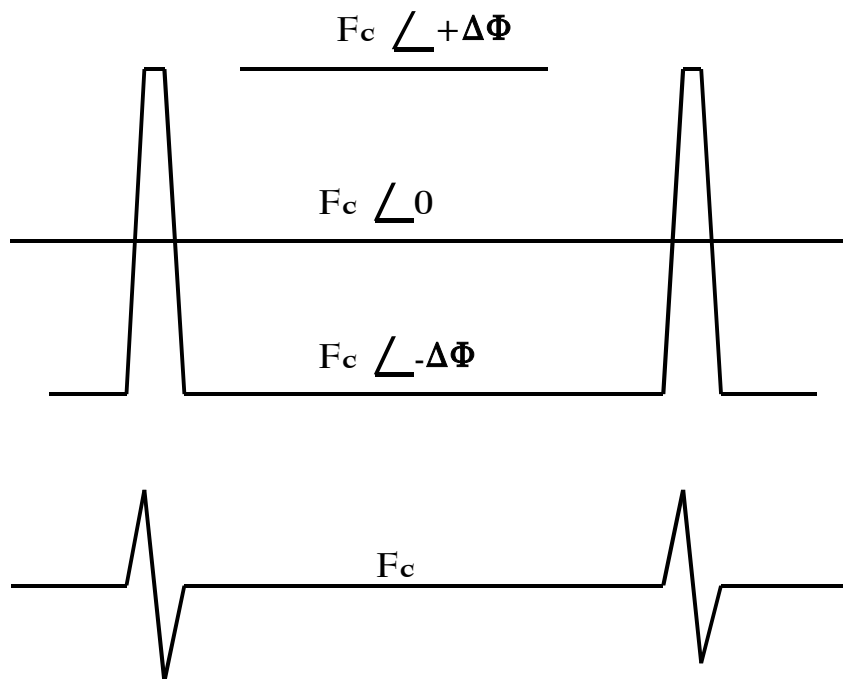


Figure 1.

Abrupt phase change digital modulation utilizes a coded baseband with abrupt edges, that is, the rise/fall times are made as abrupt, or near zero, as possible. Some RC rise time is inevitable, due to slow rates in the ICs and other parts of the circuitry. 3PRK modulation described here (Pulse Position Phase Reversal Keying) causes a phase reversal for one to three RF cycles out of a bit period. For the remainder of the bit period, the frequency and phase of the carrier are constant.

The frequency resulting from a rectangular, or burst, input is: $F = F_{\text{carrier}} + \Delta f$.

Δf can be calculated from the basic relationship $\omega t = \Phi = 2\pi f t$.

This can be rewritten in derivative form as $\Delta f = \Delta\Phi/2\pi t$.

The rise and fall time t is fixed by the the circuit parameters. During the rise and fall times, there is a large $\Delta\Phi$, which causes a large Δf of very short duration. (about 1 RF

cycle). At all other times, $\Delta\Phi$ is fixed at zero and the frequency $F = F_{\text{carrier}}$. There are no sidebands. A phase detector using F_{carrier} as a phase reference will detect the phase changes as positive and negative voltages. In practice, the phase reference is set for $F_c - \Delta\Phi$, so the the phase detected output is 180 degrees in one direction, which is seen as a phase change pulse to $F_c + \Delta\Phi$. (See Fig. 3).

This interesting characteristic of abrupt phase modulation with a rectangular input was published by Prof. Howe [1] in 1939. No use was made of the characteristic because digital phase modulation was not being used at the time, and there were no suitable filters to allow it to be utilized. These filters are now available.

If a narrow bandpass filter is used, it must have zero group delay to pass the instantaneous change in phase. It will not be broad enough to pass the instantaneous frequency changes. To all intents and purposes, there is no measurable frequency change, but there is a phase change in the carrier that is maintained constant between the rise and fall times. **The input phase change is transmitted in the single frequency of the carrier alone.**

Vectors and Bessel Products

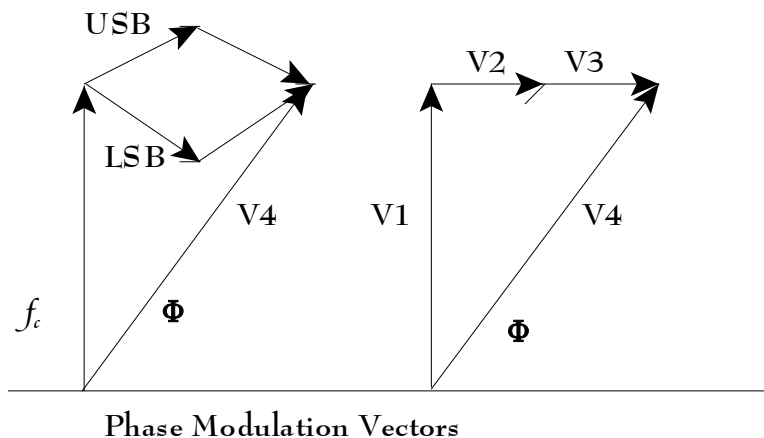


Figure 2.

According to accepted practice using PM to generate FM (Armstrong Method), a carrier and two sidebands, an upper and a lower, are required. The vectors for the upper and lower sidebands counter rotate, reaching a maximum in either direction when they are of the same phase. The upper sideband is a signal higher in frequency than the carrier by an amount equal to the modulation frequency. The lower sideband is lower in frequency by the same amount. This gives rise to Bessel products, which are necessary to cause the vector V4 to shift in phase. There are three or more different frequencies involved to produce the phase shift Φ .

When using abrupt phase modulation, the equivalents of the USB and LSB are seen as V2 and V3. They must maintain the phase shift Φ at a constant angle between phase

changes, hence they cannot rotate, but can only reverse. If they do not rotate, they are not at different frequencies, but are **at the same frequency as the carrier V1**.

Abrupt phase change angle modulation does not require any frequencies other than that of the carrier. **There are no Bessel products or other frequencies required to produce the phase shift.**

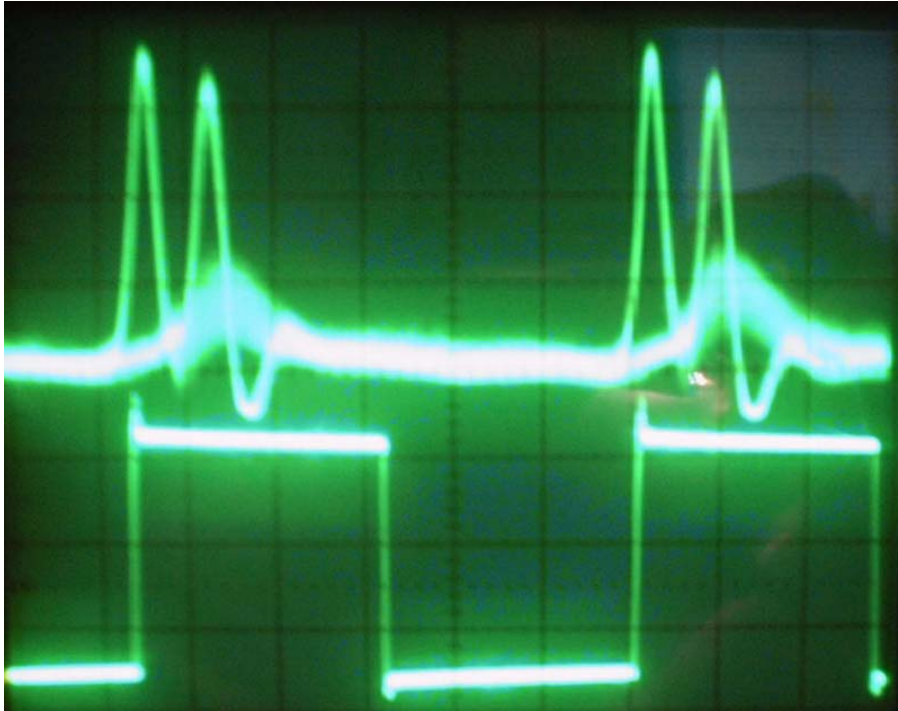


Figure 3. Detected Output Using 3PRK modulation.

Figure 3 shows the detected output from an XOR gate used as a phase detector. The reference phase is set at $F_c - \Delta\Phi$. For most of the bit period, the phase detected output is zero. At the time of the phase reversal, there is an abrupt 180 degree swing to F_c . The voltage level observed is near rail to rail for CMOS parts. See also Fig. 9.

The decoding circuitry is similar to well known pulse position decoders. The early spike is used to indicate the presence of a digital one. The delayed spike, one bit period plus a small delay, indicates a digital zero. In figure 3 the spikes are overlaid. Since the decoder is interested only in digital ones, only digital ones need be used to mark the carrier. If the spike does not fit within a timing gate for ones, a zero is assumed.

The data rate can be multiplied by using separate time periods, transmitting ones only. Figure 3 would then be two channels on the same carrier, separated by the phase reversal timing. Up to four channels with separate data streams have been successful used.

This modulation method has been used successfully on a microwave link in Denver CO, where it was added as a supplementary carrier to an existing 135 Mb/s 64QAM channel.

The data rate of the individual 3PRK channels was 1.544 Mb/s (T1). Four T1 channels were added to the main QAM channel. Newer equipment uses a 6 Mb/s rate instead of T1.

The FCC allocated channel width is 30 MHz, with a 70 MHz IF frequency. Two pulse position 3PRK channels were added at 84.997 MHz and two at 55.003 MHz.

The spectrum of an individual 3PRK channel is seen in Fig.4. It consists of a strong central frequency containing the abrupt phase change modulation and some lower level sinc/x frequencies, which are Fourier amplitude products that can be reduced or removed by filtering. They contribute nothing to the phase change. It was not necessary to reduce them at the transmitter to meet FCC rules under Part 101, since they have individual RMS levels below -80 dB compared to the main channel.

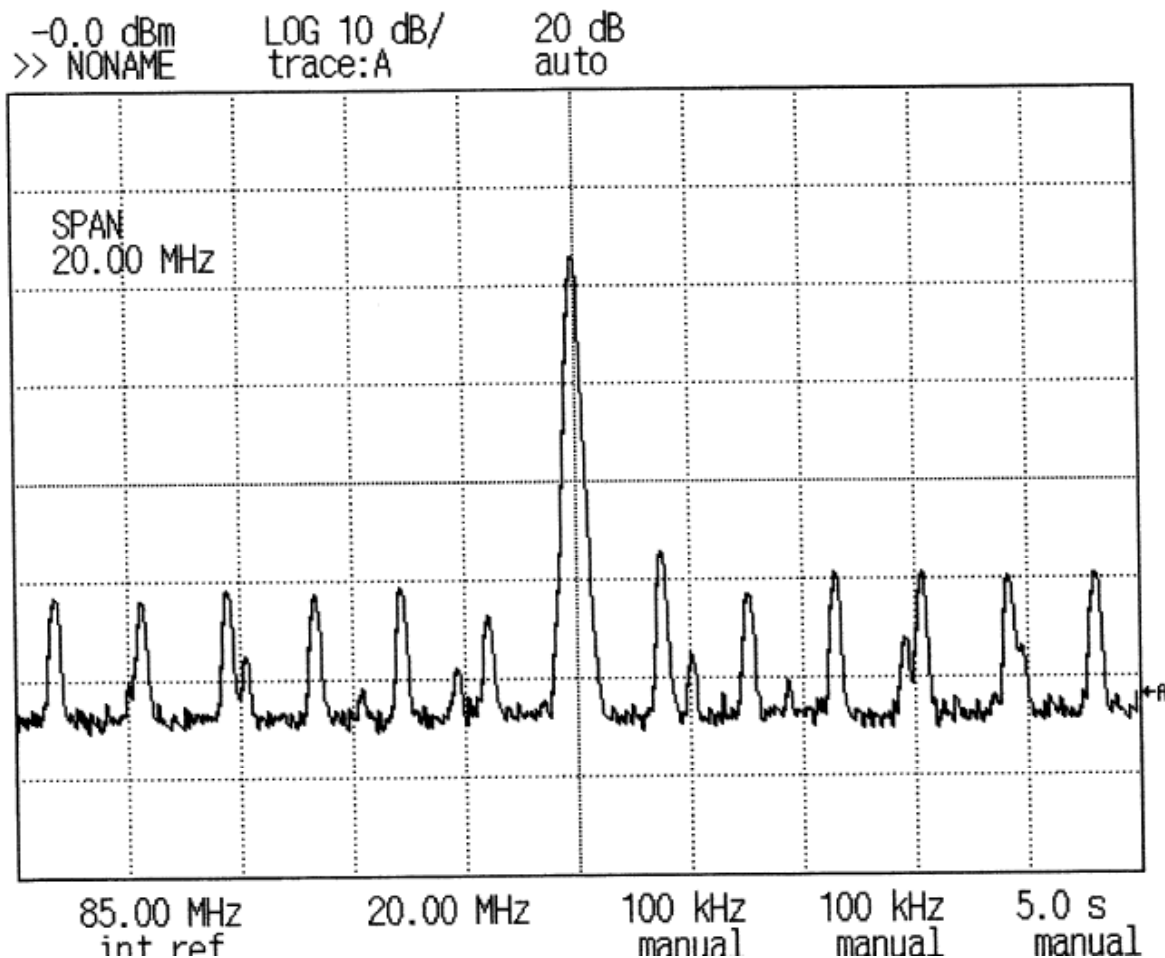


Figure 4. Spectrum of 3PRK Modulation at 84.997 MHz.

"The power spectral density and the correlation function of a waveform are a Fourier transform pair" Taub and Schilling [6], Chapter 1. Fourier products do not cause PM or FM.

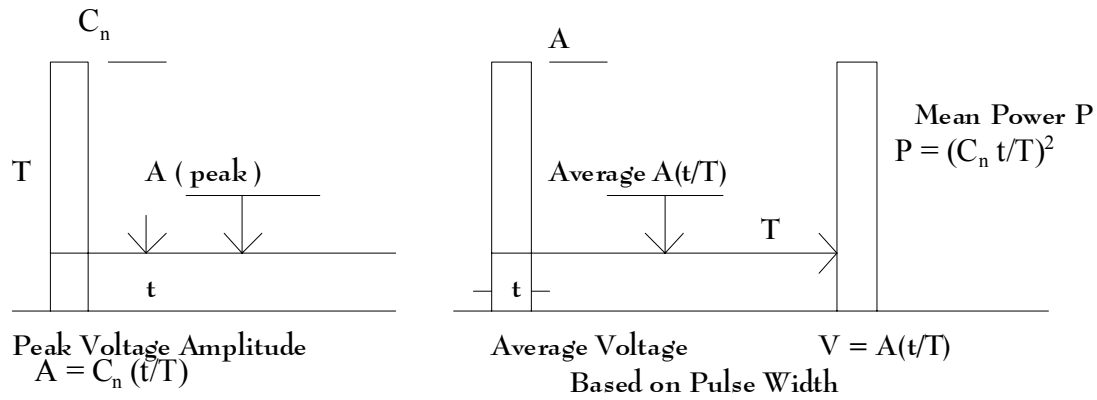


Figure 5. Peak and RMS energy in the sinc/x Fourier Spectral Components.

Fig. 6. Receiver Output Spectrum at Actual Power Levels Used. (See pp 10)

The 3PRK power level seen in Fig. 6 (page 10) is correct for a 100 kHz bandpass filter on the spectrum analyzer. The QAM power level is 23 dB higher than the 3PRK signal using a 30 MHz bandpass filter, or 13 dB lower for 4 kHz FCC filter bandwidth. 3PRK receiver boards must accept this level ± 10 dB.

The FCC measures in 4 kHz bandwidth bins. The 3PRK signal is 13 dB above the QAM in 4 kHz bins, but 23 dB below it for a 30 MHz bin.

The strongest sinc/x spikes seen in Fig. 4 are down more than 30 dB below the signal bearing frequency. The RMS energy is -60 dB. The 3PRK signal is 23 dB below the QAM signal, so the spikes are at -83 dB RMS below the main QAM signal. The FCC requires a maximum of -80 dB.

No error correction for the 3PRK channel was used in these tests. With no QAM signal present, the 3PRK signal alone measured 10^{-7} BER. With both the QAM and the 3PRK signals present, the BER remained the same. The QAM channel was employing error correction and had no measurable BER, with or without the 3PRK signals being present. There is an optimum injection level for the supplementary carriers. They should be as strong as possible without causing errors in the main channel, thereby having the main channel cause as little interference to the supplementary channel as possible. If the sinc/x peaks are 40 dB or more below the QAM 64 channel, there is no interference. In this test they were more than 50 dB below.

The secret to using this modulation method is in the zero group delay filters. If ordinary filters are used, the abrupt pulse change is smoothed over by the group delay of the filter and becomes unseen. The rise time of the filter must be able to accept the abrupt pulse change. The Walker Shunt filter makes this possible.

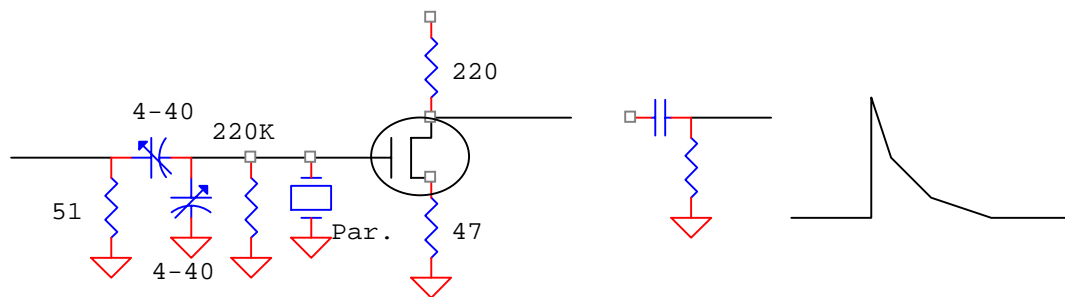


Figure 7. The Walker Shunt Filter.

The shunt filter utilizes the crystal in the parallel mode. At resonance, all the circuit reactances are cancelled and the crystal and shunting capacitor become essentially invisible. The equivalent circuit is that of a RC differentiator, or it becomes somewhat like a compensated scope probe. In the version at the left it is a resistor voltage divider. The rise time is near zero. The noise bandwidth is approximately 2-3 kHz as seen in Fig. 8. These filters can be cascaded to obtain a lower comeback on the out of band shoulders.

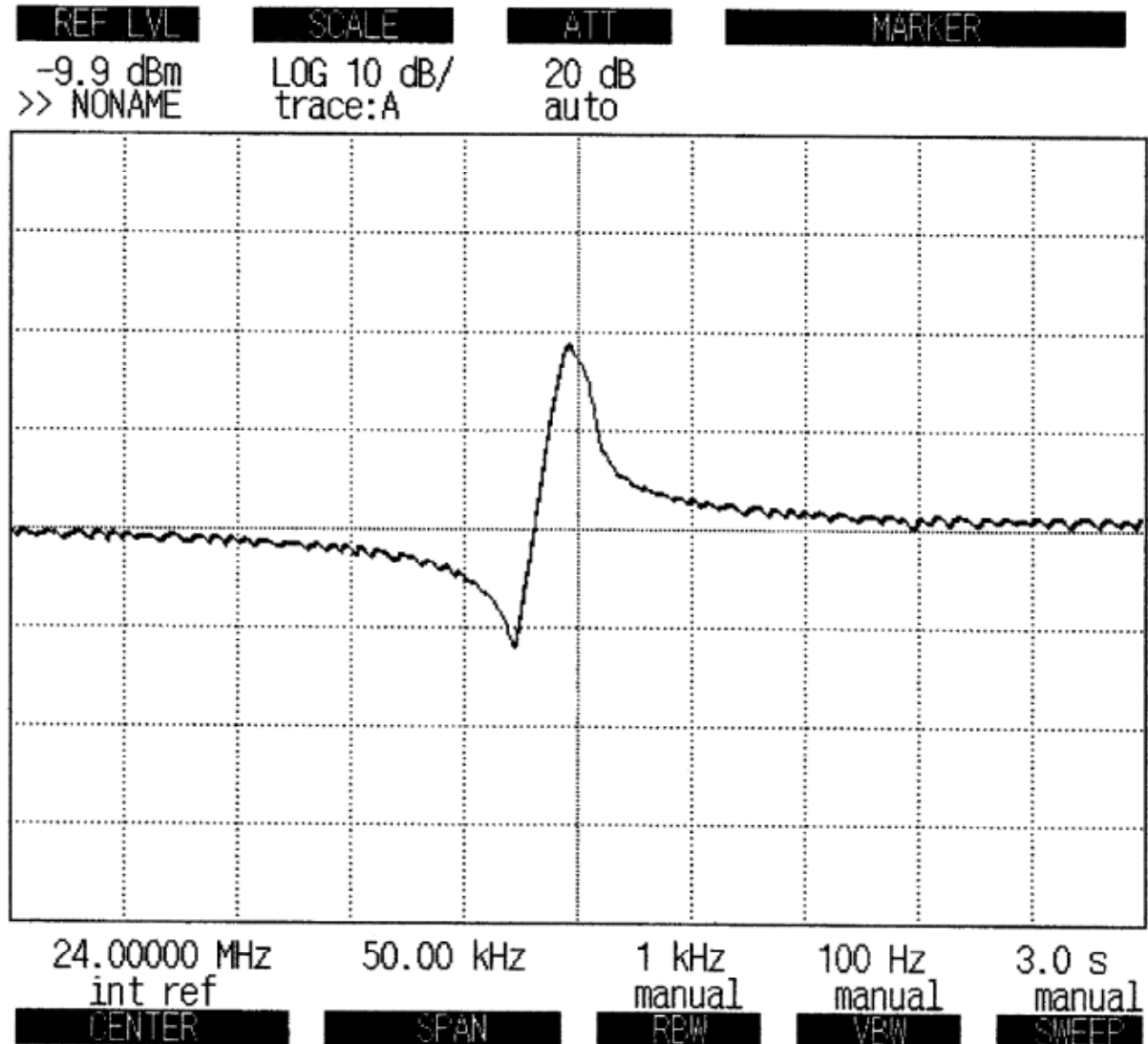


Figure 8. Bandpass Characteristic of the Walker Shunt Filter.

The shoulders of the filter are typically -17 to -20 dB below the peak. The 3 dB bandwidth is typically 2-3 kHz. Shoulder rejection as low as -30 dB has been observed. **These filters can be cascaded to obtain 50-60 dB of shoulder rejection.**

No filtering to reduce the sinc/x spikes further was used for the transmitter. FCC requirements are met without it. In the receiver, a three stage filter reduced the sinc/x spikes by an additional 55 dB. The QAM signal is obviously much stronger and this amount of filtering was required to reject the interfering components, which overlap as seen in Fig. 6.

The SNR and BER measurements for this method have been measured to be better than for BPSK modulation. This is due to the very narrow noise bandwidth of the filter. Ordinarily, the required filter bandwidth would be 1.544 MHz for a T1 signal. Since there are no Bessel sidebands with 3PRK, a filter with a noise bandwidth of 2-3 kHz is usable - if it has zero group delay. The difference in noise bandwidths accounts for the excellent SNR performance. This is discussed by Best [2].

The hardware utilized in these tests was built by Ultra Narrow Band Technologies, Inc. The path length was 14 miles with one repeater in the link.

3PRK is one embodiment of the basic modulation method, which is referred to as "Minimum Sideband", or MSB modulation. Other embodiments, along with further mathematical analyses, are discussed in the reference. [5]. MSB has been employed in practical circuits for the past three years. Several patents have been issued, with others pending.

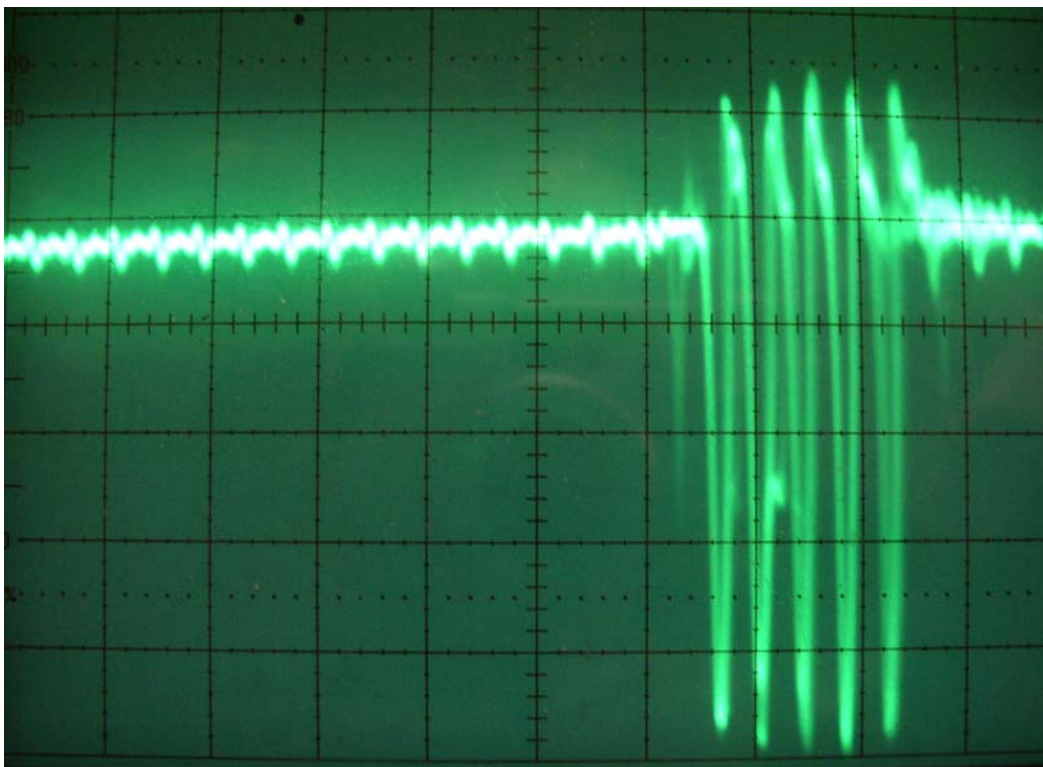


Figure 9. The detected signal using 3PRK with 5 IF cycles phase reversed, after 40 dB of sinc/x sideband reduction. The output is near rail to rail indicating the sideband reduction.

had no effect on phase change. The sampling rate is the IF frequency. The Nyquist bandwidth of the near zero group delay filter is = to the Intermediate Frequency.

Shannon's Limit:

$$R = W \text{Log}_2 (1+C/N) \text{ or as: } R = (1/\tau) \text{Log}_2 (1+C/N)$$

It is necessary to understand the meaning of W. It is not the filter noise bandwidth used, but the Nyquist bandwidth, which is and must be equal to the sampling rate. One cannot violate the Nyquist sampling theorem. The general practice is to use 1/(filter rise time) as the Nyquist bandwidth = (1/τ). Quoting Schwartz:

"The system channel capacity 'R' is obtained by multiplying the number of samples per second by the information per sample". (Schwartz, [1] pp324 and equation 6-134). The sampling rate is the intermediate frequency, or SuperBit rate. Thus W is = to the Intermediate Frequency.

It is also obvious from the hardware (and Fig. 9 above) that a data rate equal to the Intermediate Frequency can be received and decoded. The actual data rate used is lower, since multiple SuperBits (IF cycles) are used for one data bit. Assume a 48 MHz IF, then

$$48\text{MHz} = 48\text{MHz} \text{Log}_2 (1+C/N)$$

The equation will balance when C/N = 1 = 0dB. (Shannon's Limit)

Using a lower R, it appears C/N could be below 0dB as in OFSK.

Error Probability:

The error probability for any two phase (2 level) method can be calculated from:

$$P_e = \frac{1}{2} \text{erfc} [\text{SNR}]^{\frac{1}{2}} \quad (\text{Sq. root of SNR is used to get voltage ratio})$$

$$P_e = \frac{1}{2} \text{erfc} [(E_s/\eta)\tau]^{\frac{1}{2}} \quad (\text{energy ratio})$$

$$P_e = \frac{1}{2} \text{erfc} [(E_b/\eta)]^{\frac{1}{2}}$$

$$P_e = \frac{1}{2} \text{erfc} [z] \text{ where } z = V_p/1.4E_N \quad (V_p = \text{peak signal level and } E_N = \text{noise RMS})$$

Note the second of these equations. [$\tau E_{\text{signal}}/\eta$], so $\tau E_{\text{signal}} = E_b$. = signal power ON for one SuperBit period using 3 PRK. This is true energy per SuperBit. One SuperBit is being detected.

Utilizing a correlator in a post detection circuit, or in the detection circuit, the E_b can be increased by increasing τ .

$$P_e = Q \left[\frac{A}{\sqrt{Nt}} \right] = Q \left[\frac{A}{\sigma} \right] = Q \left[\frac{C\text{volts}}{N\text{volts}} \right] = \frac{1}{2} \text{erfc} \left[\frac{A}{\sigma} \right]$$

From Feher [2], where A is the peak signal value at the sampling instant and σ is the RMS voltage of the noise power at the threshold detector input. When using a cycle to cycle comparison, noise peak voltage must be compared to signal peak voltage.

$$Q_{(z)} = \frac{1}{2} \text{erfc} \left(\frac{z}{\sqrt{2}} \right) = \frac{1}{2} \text{erfc} \left(\frac{V}{\sqrt{2N}} \right) = \frac{1}{2} \text{erfc} \left(\frac{E_s}{\sigma} \tau \right)^{\frac{1}{2}}$$

Bellamy, Eq C.19, Rappaport D.11

This equation assumes a peak signal voltage V and an RMS noise voltage, which would have peaks at 1.4 times the RMS level. Using a true RMS meter, the relative peak and RMS volts are the same as measured, so the 1.4 correction is not used.

$$Q = \left(\frac{E_s}{\sigma} \tau \right)^{\frac{1}{2}} \text{ determines the BER for MSB. } \tau = 1 \text{ to } 3 \text{ for 3PRK and MCM.}$$

The theoretical and measured BER follows the Q(z) curve.

References:

- [1] Howe, Prof., As published in -- K.R. Sturley, “*Frequency Modulation*”, Chemical Publishing Co., Brooklyn, N.Y. Figure 1 was published by Prof. Howe. "Wireless Engineer", Nov. 1939. pp 547.
- [2] Best, R.E., "*Phase Locked Loops*", McGraw Hill.
Additional phase noise improvement is possible. See sect. 3.
- [3] Wm. C.Y. Lee, "*Lee's Essentials of Wireless Communications*", McGraw Hill
To paraphrase Dr. Lee, " The idea is to find ways to slightly mark the carrier wave with the modulation so that the least distortion of the carrier wave is achieved."
- [4] H.R. Walker, U.S. Pat 6,445,737 " Digital Modulation Device In A System And Method Of Using The Same". Covers 3PRK and MCM.
- [5] Further information on circuits, filters, BER measurements, etc. can be found at <http://www.VMSK.org> -
- [6] Taub and Schilling, "*Principles of Communications Systems*", McGraw Hill.
- [7] T. Rappaport, " *Wireless Communications*", Prentice Hall.
- [8] K. Feher, "*Telecommunications Measurements, Analysis and Instrumentation*", Noble Press.
- [9] J.C. Bellamy, " *Digital Telephony*", John Wiley.

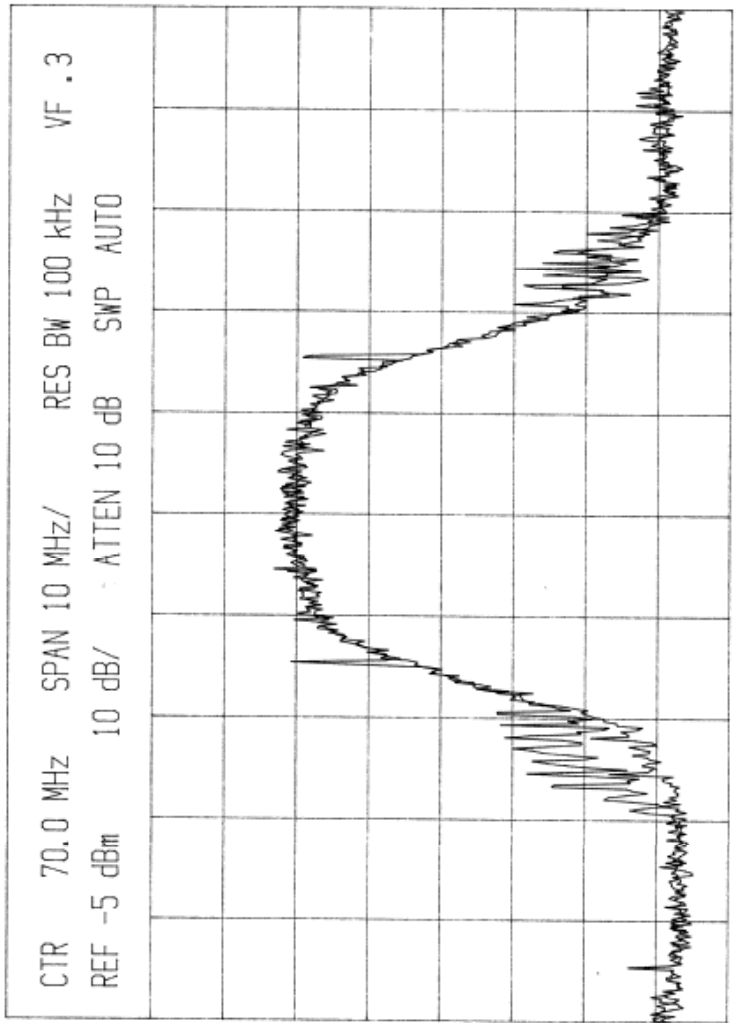


Figure 6. Spectrum of the Received Microwave QAM and 3PRK Signals at the Microwave IF amplifier Output.

The QAM signal is the central dome. The 3PRK supplementary signals are the single frequency spikes seen at the right and left sides.

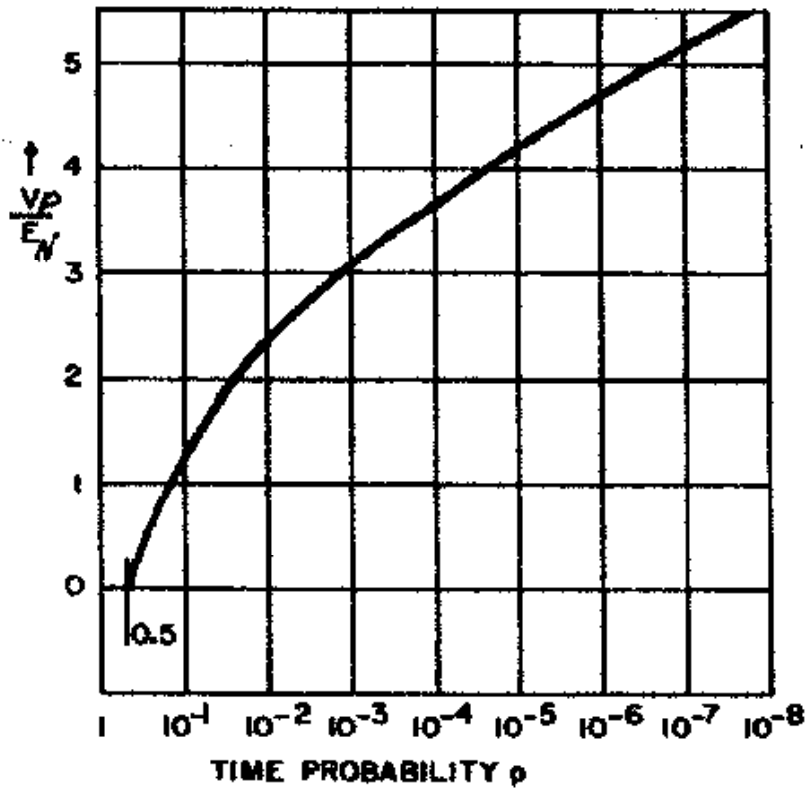


Fig. 1. Amplitude distribution of random noise. V_p is the momentary amplitude, E_n is the rms amplitude.

The probability of error follows the Q curve.

$$P(e) = \frac{1}{2} \operatorname{erfc} \left(\frac{E_s \tau}{\sigma} \right)^{\frac{1}{2}} = Q = \left(\frac{E_s \tau}{\sigma} \right)^{\frac{1}{2}}$$

